Assignment #1, due Thursday, September 6

Use Matlab for all computations. Use format long g in Matlab to see all computed digits.

1. Pick an appropriate $x_0$ and approximate $y = f(x)$ with the Taylor polynomial $p_n(x) = f(x_0) + \cdots + f^{(n)}(x_0)(x - x_0)^n/n!$ of degree $n$. Evaluate $p_n(x)$ and find an upper bound $|f(x) - p_n(x)| \leq \cdots$ using the remainder term. Find the Taylor polynomial by hand, use Matlab only for evaluating $p_n(x)$ and the error bound.

(a) $y = 32.02^{1/5}$, $n = 1$
(b) $y = \cos 0.3$, $n = 3$
(c) $y = \ln(0.97)$, $n = 1$

2. For the following examples do the following (see the example on the web page):
   (i) Find the condition number using pencil and paper. Evaluate the condition number in Matlab and print out the unavoidable error $|c_f(x)| \varepsilon_M + \varepsilon_M$ with the machine epsilon $\varepsilon_M = 10^{-16}$.
   (ii) Evaluate the formula for $y$ in Matlab machine arithmetic, yielding $\text{yhat}$. Then evaluate the formula with extra precision using vpa, yielding $\text{yextra}$. Then compute the relative error of $\text{yhat}$ as $\text{relerr} = \text{double((yhat-yextra)/yextra)}$. Compare the relative error with the unavoidable error.
   (iii) In each case there is one operation which causes the large error. Explain how much this one operation magnifies the error.
   (iv) Find a better way to compute $y$ in Matlab machine arithmetic. Compute the relative error using $\text{yextra}$. Compare the relative error with the unavoidable error.

(a) For $x = 10^{-6}$ find $y = \sqrt{9 + x} - 3$.
(b) For $x = 10^{-6}$ find $y = \ln(1 - 2x)$.
(c) For $x = 10^{-6}$ find $y = e^{2x} - e^x$. 