Structure and extinction of spherical burner-stabilized diffusion flames that are attached to the burner surface

Melvin K. Rodenhurst III, Beei-Huan Chao, Peter B. Sunderland, Richard L. Axelbaum

Department of Mechanical Engineering, University of Hawaii at Manoa, Honolulu, HI 96822, USA

Department of Fire Protection Engineering, University of Maryland, College Park, MD 20742, USA

Department of Energy, Environmental and Chemical Engineering, Washington University in St. Louis, St. Louis, MO 63130, USA

A R T I C L E   I N F O

Article history:
Received 30 May 2017
Revised 1 August 2017
Accepted 28 August 2017

Keywords:
Extinction
Spherical diffusion flame
Burner-stabilized
Premixed flame regime
Activation energy asymptotics

A B S T R A C T

The structure and extinction of a diffusion flame stabilized by a spherical porous burner, and attached to the burner, was analyzed by activation energy asymptotics. The extinction state was identified by the smallest Damköhler number, representing the weakest burning intensity, at which a flame exists. Four limiting flames, based on a fuel/air flame, but with different flow direction and inert distribution, were used to study the effects of various controlling parameters. For the attached flame, the burning characteristics and extinction state were controlled by the mass fraction of the reactant supplied from the burner (lean reactant), and the flame behaves like a premixed flame with a lean reactant. This is consistent with the premixed flame regime introduced by Liñán. With reduced Damköhler number (Da), or mass flow rate (m), the reaction becomes weaker, the leakage of the burner reactant increases, and there exists a minimum Da or m below which the flame extinguishes. Comparison of the four flames reveals that all four flames extinguish at the same fuel consumption rate when other conditions are the same. As to the effects of Lewis numbers, the flame is stronger and more difficult to extinguish when either the Lewis number of the reactant supplied from the burner is increased or the Lewis number of the reactant in the ambient is decreased. When the size of the burner is reduced, the flame reaches the burner at a smaller mass flow rate with extinction occurring also at a smaller flow rate. There exists a smallest burner size below which the flame will extinguish before reaching the burner.

© 2017 The Combustion Institute. Published by Elsevier Inc. All rights reserved.

1. Introduction

Most of our useable energy comes from burning of fossil fuels. Advanced design and operation of heat and power devices has enabled more efficient and cleaner burning systems. Although alternative energy sources have been extensively developed in recent years, production of energy through combustion processes will continue to be the primary energy source in the foreseeable future based on convenience, portability, high-energy density, steady supply, and affordable cost.

When flame extinction occurs, there can be incomplete combustion, loss of power, and formation of pollutants that may negatively affect human health. Realizing its importance, flame extinction has been studied extensively both theoretically and experimentally. The majority of these studies focused on diffusion flames because many practical combustion devises use diffusion flames for energy conversion.

A diffusion flame may be extinguished, without a loss of heat or reactants, by a kinetic limit at small Damköhler numbers, where the Damköhler number is the ratio of the flow time to the reaction time [1–11]. This kinetic extinction limit occurs when the residence time, the time allowed for the flow to pass through the reaction region, is low compared to the reaction time, and can be induced by mechanisms such as high strain rates, high scalar dissipation rates, and low reactant concentrations. The flame may also be extinguished by excessive radiative heat loss, which reduces the flame temperature and the reaction rate until the flame can no longer be sustained [12–24]. The radiative extinction limit occurs at long residence times (large Damköhler numbers) when the flame size is large. The existence of radiative extinction is unlikely to occur on earth as the intrusion of buoyancy accelerates the flow and reduces residence time.

Spherical flames have been extensively adopted to study the burning and extinction of diffusion flames because of their simplicity in geometry and the experiments can be well controlled. Traditionally, the studies were performed primarily through the burning of liquid fuel droplets in a quiescent oxidizing environment. To maintain high spherical symmetry, experiments must be
Nomenclature

\( a_{ij} \) \( j \)th order expansion of the integration constants of variable \( i \) in the outer region

\( B \) pre-exponential factor of the reaction

\( c_i \) integration constants in the reaction region

\( c_p \) specific heat of the gas at constant pressure

\( D_i \) mass diffusion coefficient of species \( i \)

\( Da \) Damköhler number

\( E \) activation temperature of the reaction

\( Le_i \) Lewis number of species \( i \)

\( m \) mass flow rate

\( m_F \) fuel consumption rate

\( q_1 \) heat of combustion per unit mass of reactant 1 consumed

\( q_F \) heat of combustion per unit mass of the fuel consumed

\( r \) spatial coordinate along the radial direction

\( r_i \) inner radius of the porous burner

\( r_b \) outer radius of the porous burner

\( r_{b,R} \) reference outer radius of the porous burner used for rescaling

\( r_f \) flame standoff location

\( T \) temperature

\( T_0 \) supplied temperature of the gas at the center of the burner

\( T_{ad} \) adiabatic flame temperature of a detached flame

\( T_{bj,i} \) \( j \)th order expansion of the temperature at the burner exit

\( T_f \) flame temperature

\( T_i \) temperature at the inner surface of the burner

\( T_{ca} \) ambient temperature

\( u \) radial flow velocity

\( W_i \) molecular weight of species \( i \)

\( Y_i \) mass fraction of reactant \( i \)

\( Y_{1,0} \) supplied value of \( Y_1 \) at the center of the burner

\( Y_{2,\infty} \) value of \( Y_2 \) at the ambient

\( \infty \) location of the ambient

Greek symbols

\( \varepsilon \) small expansion parameter defined as \( \frac{T_{ad}^2}{\rho_0} \)

\( \theta_j \) \( j \)th order expansion of the temperature in the reaction region

\( \phi_j \) \( j \)th order expansion of reactant 1 in the reaction region

\( \lambda_g \) thermal conductivity of the gas

\( \lambda_s \) thermal conductivity of the solid material used to build the porous burner

\( \tilde{\lambda} \) parameter defined as \( \psi + (\lambda_s/\lambda_g) (1 - \psi) \)

\( \Lambda \) reduced Damköhler number

\( \nu_i \) stoichiometric coefficient of species \( i \)

\( \rho \) gas density

\( \rho_f \) gas density at the flame sheet

\( \rho_{ad} \) gas density when the temperature is the adiabatic flame temperature

\( \varphi \) porosity (void space/total space) of the porous burner

\( \xi \) stretched spatial coordinate in the reaction region defined as \( (r - 1) / \varepsilon \)

\( \xi_b \) location of the burner exit in the \( \xi \) coordinate

\( \psi_j \) \( j \)th order expansion of reactant 2 in the reaction region

Superscripts

\( \sim \) nondimensional quantities

\( - \) rescaled nondimensional quantities

\( F \) fuel

\( O \) oxidizer

performed in microgravity. The kinetic extinction limit of burning droplets was first analyzed by Law [3] and the extinction Damköhler number was identified. With this information, the minimum droplet size below which a flame cannot be sustained, can be determined. The radiative extinction limit and flammability limit of droplets were later theoretically identified by Chao et al. [11]. The existence of radiative extinction was also experimentally observed in microgravity [12]. While droplet burning is able to study various flame behaviors, it can only reach quasi-steady conditions because the droplet size decreases with time after ignition, and the flow direction can only be from the fuel to the oxidizer. These difficulties may be resolved by adopting a diffusion flame stabilized by a spherical porous burner. In this system, a reactant is supplied from a porous burner and flows into a large quiescent chamber filled with the other reactant.

A burner-stabilized spherical diffusion flame can be established similar to the flame of a fuel droplet except that in the former the steady-state flame size remains unchanged when the mass flow rate is fixed. The flame size can be controlled by either varying the mass flow rate of the reactant issued from the burner or the concentration of the reactants by adjusting the type and amount of inert gas supplied with each of the reactants. In addition, the flow direction can be either from fuel to oxidizer by issuing fuel, or from oxidizer to fuel by injecting oxidizer, from the burner into the other reactant. Similar to droplet burning, experiments are best performed in microgravity to maintain spherical symmetry. Adopting the burner stabilized flames, Mills and Matalon [16] and Wang and Chao [24] have analyzed and identified the kinetic and radiative extinction limits of steady diffusion flames. Moreover, Yoo et al. [25] observed from their experimental and numerical studies that the flame might experience oscillatory motion near extinction. Propagation and radiative extinction of transient flames were also studied experimentally and numerically by Tse et al. [19], Santa et al. [21,26] and Tang et al. [22,23], where they observed that the extinction behavior is different from steady flames.

In their study of spherical burner-stabilized diffusion flames, Mills and Matalon [27] observed that the flame may reach the burner without extinction by decreasing the mass flow rate of the fuel supplied from the burner when the reaction rate, characterized by the Damköhler number, is large. An analysis for a flame attached to the burner was performed and the turning point extinction behavior was obtained. From their investigation of similar flames with radiative heat loss, and allowing the oxidizer to be supplied from the burner, Wang and Chao [24] observed that at low mass flow rates the flame may be attached to the burner before extinction even in the presence of radiative heat loss. These results were further supported by a recent numerical study by Lecoutre et al. [11] on hydrogen flames. This work revealed that in the absence of radiative heat loss, hydrogen diffusion flames contact the burner with decreasing mass flow rate, except for extremely small burner sizes. After the flame is attached to the burner, a significant amount of the reactant in the ambient is accumulated in the reaction region, in agreement with that of Mills and Matalon [27]. By continually reducing the flow rate, the flame becomes leaner (with respect to the reactant supplied from the burner) and eventually extinguishes.

The existence of a large amount of the ambient reactant in the reaction region when the flame is attached to the burner suggests that the burning is transitioned from the typical diffusion flame
regime to the premixed flame regime described by Liñán [2]. A diffusion flame is considered in the premixed flame regime when one of the reactants exhibits a significant amount in and leaking through the reaction region such that the reaction is dominated by the availability of the other (lean) reactant. Under this condition, the flame behaves like a premixed flame with its burning characteristics controlled by the lean reactant although it is a diffusion flame. The possibility of diffusion flame burning and extinction in the premixed flame regime was theoretically identified by Liñán [2], but is considered physically unrealistic and has never been observed experimentally because it is located in the middle branch of the S-shaped ignition-extinction curve. The existence of the premixed flame regime requires unusual conditions such as those of the spherical burner-stabilized flame. This study intends to extend the work by Mills and Matalon [27] by performing a more in-depth theoretical analysis of the spherical burner-stabilized flame with the flame attached to the burner exit. An analysis within the burner is included and a different approach to obtain the boundary conditions at the burner exit is taken in this study. While this flame has not been investigated experimentally because of its high flame temperature (the temperature at the exit of the burner), which is beyond what typical burners can sustain, it provides a unique feature in being able to exhibit the existence of Liñán’s premixed flame regime. Based on the flow direction and inert distribution, four limiting flames referenced to a fuel/air flame, namely the flame with (A) fuel issuing into air, (B) diluted fuel issuing into oxygen, (C) air issuing into fuel and (D) oxygen issuing into diluted fuel, have been studied. Effects of reaction rate (Damköhler number), flow rate, Lewis numbers of the reactants, residence time and the size of the burner on the burning and extinction behaviors, and the existence of Liñán’s premixed flame regime will be discussed.

2. Formulation

The problem of interest is the steady-state burning of a diffusion flame stabilized by and attached to a spherical porous burner and burning at steady state as shown schematically in Fig. 1, similar to that of Mills and Matalon [27] and Wang and Chao [24]. The burner consists of a void core region near its center in which the flow is supplied and a porous region where the flow is regulated to be uniform at its exit. The burner is considered to be spherical and gravity is assumed negligible such that the flow is uniform in the radial direction, r. Reactant 1, with a given mass flow rate m, temperature $T_0$ and mass fraction $Y_{1,0}$ is supplied from the center of the burner and flows into an infinite quiescent environment that contains reactant 2 at a temperature of $T_\infty$ and mass fraction $Y_{2,\infty}$. Combustion is described by a one-step, overall and irreversible reaction with a rate that follows a second order Arrhenius kinetics (first order with respect to each of the reactants) with high activation energy. Since this work intends to study the kinetic limit of the flame, heat loss is not considered.

The burner is considered operating without cooling. The burner material and the gas are assumed to be in thermal equilibrium within the burner such that their temperatures are the same at any given radial location. The equivalent thermal conductivity in the burner is then given by $\phi \lambda_s + (1 - \phi) \lambda_s$. All the notations are defined in the Nomenclature section. Applying the above assumptions and problem description, the mass conservation equation yields a constant mass flow rate given by $m = 4\pi r^2 \rho u \phi$ in the porous region and $4\pi r^2 \rho u \phi$ elsewhere. Penetration of the ambient reactant into the burner is considered negligible such that $Y_1 = Y_{1,0}$ and $Y_2 = 0$ before the flow exits from the burner, while the energy conservation equation in the core and burner regions in a nondimensional form is

\begin{equation}
\frac{d}{d\tilde{r}} \left( \tilde{m} \tilde{T} - \tilde{r}^2 \frac{d \tilde{T}}{d\tilde{r}} \right) = 0, \quad 0 < \tilde{r} < \tilde{r}_i, \tag{1}
\end{equation}

\begin{equation}
\frac{d}{d\tilde{r}} \left( \tilde{m} \tilde{T} - \tilde{r}^2 \tilde{\lambda} \frac{d \tilde{T}}{d\tilde{r}} \right) = 0, \quad \tilde{r}_i < \tilde{r} < 1. \tag{2}
\end{equation}

In the gas region outside of the burner, the energy and species conservation equations are

\begin{equation}
\frac{d}{d\tilde{r}} \left( \tilde{m} \tilde{T} - \tilde{r}^2 \tilde{\lambda} \frac{d \tilde{T}}{d\tilde{r}} \right) - \tilde{r}^2 Da \tilde{\beta}^2 \tilde{Y}_1 \tilde{Y}_2 \exp (-\tilde{\phi}/\tilde{T}), \tag{3}
\end{equation}

\begin{equation}
\frac{d}{d\tilde{r}} \left( \tilde{m} \tilde{\lambda} \frac{d \tilde{Y}_1}{d\tilde{r}} \right) = \frac{d}{d\tilde{r}} \left( \tilde{m} \tilde{\lambda} \frac{d \tilde{Y}_2}{d\tilde{r}} \right) = 0, \quad \tilde{r}_i < \tilde{r} < 1. \tag{4}
\end{equation}

Eqs. (1)–(4) are to be solved subject to the boundary and interface conditions

\begin{equation}
\tilde{r} = 0 : \quad \tilde{T} = \tilde{T}_0, \tag{5}
\end{equation}

\begin{equation}
\tilde{r} = \tilde{r}_i : \quad \tilde{T} = \tilde{T}_i, \quad (d \tilde{T}/d\tilde{r})_{\tilde{r}_i} = \tilde{\lambda} (d \tilde{T}/d\tilde{r})_{\tilde{r}_i}. \tag{6}
\end{equation}

\begin{equation}
\tilde{r} = 1 : \quad \tilde{T} = \tilde{T}_b, \quad \tilde{\lambda} \left( \frac{d \tilde{T}}{d\tilde{r}} \right)_{\tilde{r}_i} = \left( \frac{d \tilde{T}}{d\tilde{r}} \right)_{\tilde{r}_i}, \tag{7}
\end{equation}

\begin{equation}
\tilde{r} \rightarrow \infty : \quad \tilde{T} \rightarrow \tilde{T}_\infty, \quad \tilde{Y}_1 \rightarrow 0, \quad \tilde{Y}_2 \rightarrow \tilde{Y}_2. \tag{8}
\end{equation}

where $\tilde{T}_0$ and $\tilde{T}_b$ are to be determined from the analysis. In the above, the nondimensional variables and parameters are defined as

\begin{equation}
\tilde{T} = \frac{c_p T}{q_1 Y_{1,0}}, \quad \tilde{Y}_1 = \frac{Y_1}{Y_{1,0}}, \quad \tilde{Y}_2 = \frac{Y_2}{Y_{2,0}}, \quad \tilde{m}_i = \frac{c_p m}{4\pi \rho_f T_b \lambda_s}, \quad \tilde{\lambda} = \phi + (1 - \phi) \lambda_s/\lambda_s,
\end{equation}

\begin{equation}
\tilde{r} = \frac{r}{r_b}, \quad \tilde{\phi} = \frac{c_p E}{q_1 Y_{1,0}}, \quad \tilde{\rho} = \frac{\rho}{\rho_f}, \quad \tilde{L}_e = \frac{\lambda_s/c_p}{\rho D_i}, \quad Da = \frac{v_2 W_2 c_p B \rho_f^2 r_b^2 Y_{1,0}}{\lambda_s}. \tag{9}
\end{equation}
The values of $c_p$, $\lambda_\infty$, $\lambda_\Lambda$, $\lambda_\Delta$ are assumed to be constants. Eqs. (1) and (2) can be integrated subject to the boundary and interface conditions in Eqs. (5) and (6) to yield [24]

$$\tilde{T} = \tilde{T}_0 + \left( \tilde{T}_b - \tilde{T}_0 \right) \exp \left[ \left( \tilde{m}/\tilde{\lambda}_1 \right) (1 - \tilde{T}^{-1}) \right] \exp \left[ \tilde{m} (\tilde{T}^{-1} - 1) \right], \quad 0 < \tilde{T} \leq \tilde{T}_I.$$  

In the process, $\tilde{T}_I$ has been solved in terms of $\tilde{T}_b$.

For a sufficiently high mass flow rate, a flame is established outside of the burner after ignition. The flame, defined as the region in which significant combustion reaction occurs, is spherical and situated near the boundary where the stoichiometric reaction between the two reactants is attained. Only small amounts of reactant may leak through the flame for a diffusion flame. For this detached flame, the flame stand-off distance and the flame temperature can be determined following the works of Mills and Matalon [27] and Wang and Chao [24] as $\tilde{T}_f = L_{\text{ed}} \tilde{m}/\tilde{\epsilon} n (1 + \tilde{Y}_2, \infty)$ and

$$\tilde{T}_f = 1 + \tilde{T}_0 - \left( (1 + \tilde{T}_0 - \tilde{\epsilon}) / (1 + \tilde{Y}_2, \infty) \right).$$

When $L_{\text{ed}} = 1$, the flame temperature is the adiabatic flame temperature,

$$\tilde{T}_{ed} = 1 + \tilde{T}_0 - \left( (1 + \tilde{T}_0 - \tilde{\epsilon}) / (1 + \tilde{Y}_2, \infty) \right).$$

By decreasing the mass flow rate, the flame moves towards the burner and is extinguished if the reaction is relatively weak. For a flame with sufficiently strong reaction, it eventually attaches to the burner surface when $\tilde{T}_f = 1$ or $\tilde{m} = \tilde{n} n ((1 + \tilde{Y}_2, \infty) / L_{\text{ed}}$. Because of the intrusion of the burner, the flame stays at the burner exit for all flow rates below this critical flow rate. The reduction of the flow rate from the critical rate reduces the supply of reactant 1 and allows a large amount of reactant 2 to accumulate in the reaction zone and reach the burner. As a result, the reaction is controlled by the availability of reactant 1 (the lean reactant), similar to that of a premixed flame with a lean reactant such that the burning characteristics transition from the typical diffusion flame to a reaction which is similar to that of a premixed flame [2]. This study focuses on the attached flame for which $\tilde{m} < \tilde{n} n ((1 + \tilde{Y}_2, \infty) / L_{\text{ed}}$.

In the regions away from the reaction region (outer regions), chemical reaction is negligible because of the low gas temperature. For the attached flame, there is only one outer region in the ambient side of the reaction region. In this outer region, solving the source-free expression of Eqs. (3) and (4) subject to Eq. (8) gives the outer solutions

$$\tilde{T} = \tilde{T}_\infty + \left[ a_{T,0} - \tilde{m} a_{T,1} + O (\tilde{m}^2) \right] [1 - 1 - (\tilde{m}/\tilde{\epsilon})],$$

$$\tilde{Y}_1 = [a_{T,0} + \tilde{m} a_{T,1} + O (\tilde{m}^2)] [1 - 1 - (\tilde{m}/\tilde{\epsilon})],$$

where $\tilde{\epsilon}$ is the small expansion parameter. In addition, the temperature at the burner exit is expanded as $\tilde{T}_b = \tilde{T}_{b,0} - \tilde{m} \tilde{m}/\tilde{\epsilon} n (1 + \tilde{Y}_2, \infty) / L_{\text{ed}}$ such that there is a significant, i.e., an O(1) amount of reactant 2 in the $O(\tilde{m})$ reaction region located near $\tilde{T} = 1$. The study is not applicable to the cases when $\tilde{m}$ is only slightly smaller than $\tilde{n} n ((1 + \tilde{Y}_2, \infty) / L_{\text{ed}}$ for which the amount of reactant 2 is still an O(\tilde{m}) quantity in the reaction region.

In the reaction (inner) region, a stretched spatial coordinate is defined as $\xi = (\tilde{T} - 1) / \tilde{\epsilon}$, where $\tilde{\epsilon} = \tilde{T}_{b,0} / \tilde{\epsilon} n$, and the variables are expanded as $\tilde{\rho} = 1 + O (\tilde{\epsilon})$.

$$\tilde{T} = \tilde{T}_{b,0} - \tilde{m} \theta_1 - \tilde{m} \theta_2 + O (\tilde{\epsilon}^2),$$

$$\tilde{Y}_1 = \theta_1 + \tilde{m} \theta_2 + O (\tilde{\epsilon}^2),$$

$$\tilde{Y}_2 = \theta_2 + \tilde{m} \theta_1 + O (\tilde{\epsilon}^3).$$

For the high activation energy reaction, only an $O(\tilde{m})$ variations in the variables is allowed so that $\theta_2$ is a constant and the leading order flame temperature is $\tilde{T}_f = \tilde{T}_{b,0}$. Substituting these inner expansions into Eqs. (3) and (4), expanding in terms of $\tilde{\epsilon}$, and keeping the leading order terms, we obtain the inner equations

$$d^2 \theta_1 / d \xi^2 - \Lambda \theta_1 \exp (-\theta_1),$$

$$d (\theta_1 - L_{\text{ed}}^{-1} \theta_1) / d \xi = c_1,$$

$$d (L_{\text{ed}}^{-1} \theta_1 - L_{\text{ed}}^{-1} \theta_2) / d \xi = c_2,$$

$$[d (\theta_2 - L_{\text{ed}}^{-1} \theta_2) / d \xi] - \tilde{m} \theta_1 - \phi_1 = c_3 - c_2 \xi.\]$$

where $\Lambda = \tilde{\epsilon}^2 D a \tilde{n} \exp (-\tilde{\epsilon} / \tilde{T}_{b,0})$ is the reduced Damköhler number.

Part of the boundary conditions required to solve Eqs. (19)-(22) can be determined from matching the inner and outer solutions as $\xi \to \infty$. Through matching, we have $a_{T,0} = 0, a_{T,1} = 0, a_{T,2} = 0, \phi_1 = 0$ and $\phi_1 = 0, a_{T,1} = 0, \phi_1 = \tilde{m} \theta_1$. Eqs. (23a) and (24a) also imply

$$d (\theta_1 / d \xi) \big|_{\xi = \infty} = \phi_1 = 0.$$  

The other group of the conditions is obtained from equating the inner solutions with the interface conditions at the burner exit shown in Eq. (7). When the flame first reaches the burner, the burner is located at $\tilde{T} = 1$ with the burner exit at $\tilde{\xi} = \infty$, and the flame is a typical diffusion flame with an $O(\tilde{m})$ leakage for both reactants. Additional reduction in mass flow rate results in the flame shifting further towards the burner such that the burner is located at a finite distance $\tilde{\xi} = \tilde{\xi}_b$. The boundary conditions at the burner exit are then

$$\theta_1 = 0,$$

$$d (\theta_1 / d \xi) \big|_{\xi = \tilde{\xi}_b} = -\tilde{m} \theta_1,$$

$$d (\phi_1 / d \xi) \big|_{\xi = \tilde{\xi}_b} = -\tilde{m} \phi_1,$$

$$d (\phi_1 / d \xi) \big|_{\xi = \tilde{\xi}_b} = \phi_1 = 0.$$
Mills and Matalon [27] adopted $\xi = 0$ at the burner exit, which yields different boundary conditions. Applying Eqs. (23b), (24b), (25)–(27) and (29)–(32) to Eqs. (20)–(22) yields $\psi v = 1 + \bar{t}_0 - \bar{t}_\infty$, $a_{r,0} = 1 + \bar{y}_\infty$, and
\begin{equation}
\psi_0 (1 + \bar{y}_\infty) \exp \left(-L_y \bar{m} \right) - 1.
\end{equation}
(33)

$T_b,0 = 1 + \bar{t}_0 - (1 + \bar{t}_0 - \bar{t}_\infty) \exp(-\bar{m})$.
(34)

$a_{r,1} = a_{r,0} = 2 (1 + \bar{t}_0 - \bar{t}_\infty) \exp \left(-\bar{m} \right)$.
(35)

Next, integrating Eq. (20) once subject to Eqs. (23a) and (24a), we obtain a relation between $\theta_1$ and $\phi_1$,
\begin{align}
\phi_1 = \alpha_{r,1} \left[ 1 - \exp (-L \bar{m}) \right] + L \bar{m} \left[ \theta_1 - a_{r,1} \left[ 1 - \exp (-\bar{m}) \right] \right] - (1 + \bar{t}_0 - \bar{t}_\infty) \bar{m} \exp \left(-\bar{m} \right) \frac{\bar{m}}{}.
\end{align}
(36)

With the application of Eq. (36), Eq. (19) can be integrated numerically subject to the four boundary conditions expressed in Eqs. (23a), (23b), (28) and (29) to complete the analysis. A fourth order Runge–Kutta method was adopted in this work. As only two conditions are required to integrate Eq. (19), the other two conditions, along with Eq. (35), are used to determine $T_b,0$, $\bar{t}_b,0$, $\alpha_{r,1}$ and $a_{r,1}$. Because there are four variables to be determined by three conditions, an additional condition is necessary, as observed in the analyses of some premixed flames. Following that of Law et al. [28], the last condition is obtained by letting the outer solution of $\bar{t}$ be the same as $\bar{t}_b$ to $O(\varepsilon)$, which results in $\bar{t}_b,1 = \alpha_{r,1} \left[ 1 - \exp (-\bar{m}) \right]$.

3. Results and discussion

To present the results of this study, sample numerical computations were performed using the thermal physical data that are closely mimic ethylene/air flames, given by
\begin{align}
T_0 &= T_\infty = 298 K, \quad q_r = 47, 160 J/g, \quad c_p = 1.3232 J/(g K)
\end{align}
\begin{align}
\lambda_g &= 0.0012043 W/(cm K)
\end{align}
\begin{align}
v_f &= 1, \quad v_0 = 3, \quad W_f = 28 g/mole, \quad W_0 = 32 g/mole, \quad E = 24,000 K.
\end{align}

Assuming complete combustion, the adiabatic flame temperature corresponding to these data is $T_{ad} = 2566 K$. Except for the last figure, all other calculations were performed using $r_b = 0.3175 cm$.

As described in the introduction section, both the flow direction and inert distribution can be independently varied for the burner-stabilized flame. With this flexibility, and using a fuel/air mixture as a reference, where air consists of 21% oxygen and 79% nitrogen on a molar basis, the four fuel/oxygen/nitrogen flames that have been studied in the past [24,26,29,30] are studied in this work. These flames are: (A) fuel issuing into air, (B) diluted fuel issuing into oxygen, (C) air issuing into fuel and (D) oxygen issuing into diluted fuel. The flow direction is from fuel to oxidizer for Flames A and B, and from oxidizer to fuel for Flames C and D. Flames B and D are obtained by extracting the inert from the air of Flames A and C and diverting it to the fuel in such a way that all flames have the same adiabatic flame temperature but different residence time.

Recognizing that the values of parameters used for nondimensionalization are different for the four limiting flames, the same nondimensional value represents different physical values. The interpretation could be misleading when comparing the results of these flames. To avoid any misconceptions, the parameters are rescaled to be independent of flow condition, burner size and flame type by defining
\begin{align}
\bar{T} &= \frac{c_p T}{\bar{E}}, \quad \bar{Y}_1 = \frac{Y_1}{\bar{E}}, \quad \bar{E} = \frac{c_p E}{\bar{Y}_1}, \quad \bar{m} = \frac{c_p m}{\bar{Y}_1}, \quad \bar{E} = \frac{\bar{T}^2}{\bar{E}}, \quad \bar{D} = \frac{\bar{m}}{\bar{E}^2 \bar{D} a} \exp (\bar{E}/\bar{T}_{ad}).
\end{align}

As a result, the parameter representing the leakage of the reactant supplied from the burner is rescaled to $\bar{a}_{1,1} = \bar{Y}_1(\bar{E}/\bar{E}) \bar{a}_{1,1}$. The value of $r_b,1$ is chosen as 0.3175 cm in this work. Based on the stoichiometric relation, the fuel consumption rate is $\bar{m}_g = \bar{m}$, $\bar{m}_f = 0.129, \bar{m}_f = 0.171$ and $\bar{m}_f = 0.434$ for Flames A, B, C and D, respectively. Applying the specified thermal physical data, the flame first reaches the burner at $\bar{m}_f = 0.0658, 1.52, 2.75$ and 0.246, or $\bar{m}_f = 0.0658, 0.124, 0.187$ and 0.0718, for these flames. The present study is valid only for flow rates sufficiently lower than these values.

Discussion of the results starts from plotting the leakage of the reactant supplied from the burner, $\bar{a}_{1,1}$, versus the reduced Damköhler number, $\bar{A}$, for selected values of the mass flow rate, $\bar{m}$, as shown in Fig. 2 for the four flames. The Lewis numbers of both of the reactants are set to be unity in these calculations. The results show the typical lower and middle branches of the S-shaped ignition-extinction curve with respect to reactant leakage. That is, for a given $\bar{m}$, there are two solutions corresponding to a value of $\bar{A}$ when the reaction is sufficiently strong. By decreasing the value of $\bar{A}$ (weakening the reaction), the two solutions approach each other. There exists a minimum $\bar{A}$ at which the two solutions merge to one and there is no solution below this critical value. This minimum value of $\bar{A}$, below which a solution does not exist, is identified as the extinction limit. The lower branch of the curve, shows a small amount of reactant leakage with the leakage increasing with decreasing reaction rate, and this is the physically realistic solution.

For each of the flames, the flame supported by a lower $\bar{m}$ has a larger amount of reactant leakage for the same value of $\bar{A}$ and is extinguished at a larger value of $\bar{A}$, meaning that it is easier to be extinguished. As shown in Eq. (34), the flame with a smaller $\bar{m}$ has a lower flame temperature because of a reduced availability of reactant 1 (the reactant supplied from the burner), which is similar to lean premixed flame burning (with respect to reactant 1). The reaction rate is a strong function of flame temperature because of the high activation energy. Unlike the detached flame for which the flame temperature depends on the Lewis number of the ambient reactant as shown in Eq. (11), the flame temperature of an attached flame is independent of the Lewis numbers, as shown in Eq. (34). It only depends on the availability of reactant 1 via $\bar{m}_f$, again similar to that of a 1-dimensional premixed flame.

While the above discussion is informative in exhibiting the burning and extinction behaviors, it is more realistic experimentally to keep the reduced Damköhler number fixed and vary the flow rate. In Fig. 3 the leakage of the burner reactant is plotted versus the mass flow rate for specified values of $\bar{A}$ for the four flames. The results show that for each given $\bar{A}$, the reaction is weaker by reducing $\bar{m}_f$ because the reaction is leaner with respect to reactant 1, and there exists a minimum mass flow rate below which the flame extinguishes. This is in agreement with that of Fig. 2 and Mills and Matalon [27].

Comparison of the four flames can be made by plotting the mass flow rate, $\bar{m}_f$, versus the reduced Damköhler number, $\bar{A}$, at the extinction states as shown in Fig. 4 in dashed curves. Because of the difference in $\bar{m}$ for these flames, it is difficult to make comparisons. By plotting the results using the fuel consumption rate, $\bar{m}_f$, it is observed that the extinction states are very close to each other for all four flames. That is, when the reduced Damköhler number is the same, all flames extinguish at practically the same fuel consumption rate regardless of flow direction, inert distribution (stoichiometric mixture fraction) or residence time. This is not
expected a priori since the flow direction and inert distribution have strong effects on the extinction of the detached flames [24]. The results indicate that the burning characteristics and extinction of attached flames are controlled only by the availability of the reactant supplied from the burner (lean reactant), as expressed by their equivalent fuel consumption rate, regardless of how the reactants enter the reaction region. This behavior appears to be similar to that of a premixed flame, as described by Liñán for the premixed flame regime [2].

The effect of the Lewis number of the ambient reactant (reactant 2) on the flame behavior is shown in Fig. 5 for Flame A by plotting the leakage of the reactant supplied from the burner (reactant 1) against the mass flow rate for a specified value of $\bar{\Lambda} = 400$, $Le_1 = 1$ and selected values of $Le_2$. The results of the other three flames are not shown because the qualitative behaviors of the four flames are the same, as shown in Figs. 2 and 3. The figure shows that the flame is extinguished at smaller flow rate, meaning that the burning intensity is stronger, for lower values of $Le_2$. While this result is qualitatively similar for both the detached and attached flames, the mechanisms are different. For a detached flame, a decrease in $Le_2$ yields a higher flame temperature, as shown in Eq. (11), and hence a greater reaction rate. As to the attached flame, the flame temperature is independent of $Le_2$ as mentioned earlier. When $Le_2$ is decreased, the mass diffusion rate of reactant 2 is higher and a larger amount of the ambient reactant is transported to the reaction region, as shown in Eq. (33). The
higher amount of reactant 2 facilitates the consumption of reactant 1, which results in a reduction in leakage, and delays extinction.

As a counterpart of the previous discussion, the effect of the Lewis number of the reactant supplied from the burner (reactant 1) on the flame behavior is shown in Fig. 6 for Flame A by plotting the leakage of reactant 1 versus the mass flow rate for $\bar{\Lambda} = 400$, $Le_2 = 1$ and selected values of $Le_1$. Contrary to the effect of $Le_2$, the flame is stronger and extinguishes at smaller flow rates for higher values of $Le_1$. Like the effect of $Le_2$, this behavior is qualitatively similar to that of the detached flame which was studied by Mills and Matalon [27]. Physically, by decreasing the value of $Le_1$, the mass diffusion rate of reactant 1 is higher, which renders the reactant passing through the reaction region at a higher speed and the residence time for reaction is reduced. As a consequence, more of reactant 1 escapes from the reaction region such that the reactant leakage is increasing and the flame becomes weaker and is more vulnerable for extinction.

The flame response to the size of the burner, $r_b$, is shown in Fig. 7 for Flame A by plotting the leakage of reactant 1 versus the mass flow rate for $\bar{\Lambda} = 600$, $Le_1 = Le_2 = 1$ and a few values of $r_b$. Figure 7 shows that by reducing the burner size, the flame extinguishes at a smaller mass flow rate. When a smaller burner is used, the flame is detached from the burner until a smaller mass flow rate is issued. Therefore, the transition from the detached flame to the attached flame occurs at a smaller flow rate for a smaller burner, and the extinction is delayed. The use of a smaller burner also yields an increase in flow velocity at the exit of the burner. Stronger convection further increases the difficulty for the flame to reach the burner. Ultimately, there exists a minimum burner size below which the flame cannot be attached to the burner. That is, extinction occurs before the flame is able to reach the burner, like the detached flames previously studied [16,24,27]. The minimum burner size to obtain an attached flame is $r_b = 0.05038$ cm for the present calculations. These results qualitatively agree with the numerical study of a hydrogen/air flame by Lecoustre et al. [11].

To maintain spherical symmetry, experiments can only be performed in microgravity for all four flames. While the present analysis keeps the composition of the unreacted fuel unchanged for Flames C and D, it is understood that the fuel will be decomposed to lighter molecules such as CO and H$_2$. Results for these two flames may be obtained by measuring the amount of equiva-
lent carbon and/or hydrogen fractions that are not burnt to carbon dioxide and water.

4. Concluding remarks

In this research, the burning and extinction characteristics of a spherical diffusion flame stabilized by and attached to the exit of a spherical porous burner were analyzed using activation energy asymptotics. Four limiting flames were investigated using a fuel/air flame as a reference, and all flames had the same adiabatic flame temperature but different inert distribution, flow direction and residence time. The effects of reaction rate, flow direction, inert distribution (flame structure), residence time, Lewis number of the reactants and burner size on flame extinction were investigated. The flow direction was either from the fuel to the oxidizer or from the oxidizer to the fuel, and the inert was supplied either with the oxidizer or the fuel.

For any of the four flames with constant mass flow rate, flame extinction occurs by lowering the reaction rate (Damköhler number). When the reaction rate is fixed, a reduction in the mass flow rate results in a weaker flame because of the reduced supply of burner reactant, and there exists a minimum flow rate below which extinction occurs. Comparison of the four flames shows that all of these flames extinguish at nearly the same fuel consumption rate when they have the same Damköhler number. A decrease in Lewis number of the ambient reactant results in the flame being more difficult to extinguish because of an increased availability of ambient reactant in the reaction region. On the contrary, the flame is easier to extinguish when the Lewis number of the burner reactant is reduced because the burner reactant passes through the reaction zone at a faster rate such that the residence time is reduced. By using a smaller burner, the flame reaches the burner and is extinguished at a smaller flow rate. There exists a minimum burner size below which the flame cannot reach the burner because the flow velocity is too high.

When a flame is attached to the burner, the reaction is dominated by the availability of reactant supplied from the burner and the flame behaves like a premixed flame lean with respect to the burner reactant. This is consistent with the characteristics of a diffusion flame when it is burning in the premixed flame regime introduced by Liñán [2]. As a result, this study, as well as earlier studies by Mills and Matalon [27] and Lecoustre et al. [11], suggests the possibility of diffusion flames burning in the premixed flame regime.

Acknowledgments

This research was supported by the National Aeronautics and Space Administration under grants NNX11AQ90A (BHC), NNX15AB60A (PBS) and NNX15AC75A (RLA) with D.P. Stocker as grant monitor. The authors thank Q. Crowell for his assistance with part of the numerical computations.

References