Optimum dilution rate for cell/product productivity -- /w Substrate Inhibition, Vary both D and s_f.
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Model parameters:

$\mu_m := 0.7 \quad K := 0.02 \quad K_i := 0.1 \quad Yx := 0.5 \quad Yp := 0.15 \quad \alpha := 0.1 \quad \beta := 0.02$

$$\mu(s) := \frac{\mu_m \cdot s}{K + s + K_i \cdot s^2}$$

Initial guesses:

$x := 1 \quad s := 0 \quad p := 0$

Given Steady-state equations:

dx/dt: $0 = (\mu(s) - D)\cdot x$

ds/dt: $0 = D\cdot(s_f - s) - \frac{1}{Yx}\cdot\mu(s)\cdot x - \frac{1}{Yp}\cdot(\alpha\cdot\mu(s)\cdot x + \beta\cdot x)$

dp/dt: $0 = \alpha\cdot\mu(s)\cdot x + \beta\cdot x - D\cdot p$

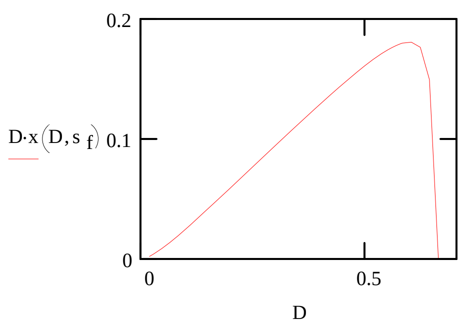
$ans(D, s_f) := Find(x, s, p)$ ← Allow both D and s_f to be optimized. An example: $ans(0.1, 1) = \begin{pmatrix} 0.249 \\ 0.003 \\ 0.075 \end{pmatrix}$

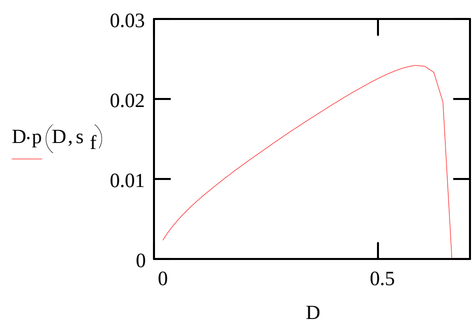
$x(D, s_f) := ans(D, s_f)_0 \qquad p(D, s_f) := ans(D, s_f)_2$

$w_x := 0.01 \quad w_s := 0.04$

Max $\quad Profit(D, s_f) := D\cdot p(D, s_f) - w_x\cdot D\cdot x(D, s_f) - w_s\cdot D\cdot s_f$

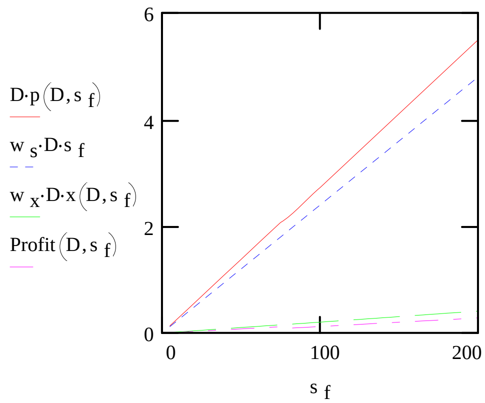
Dependence of D·p and D·x on D: $\quad D := 0.02, 0.04 .. 0.699 \qquad s_f := 1$





Note that productivity is the highest near $D=\mu_m$, but drops off very quickly at $D=\mu_m$.

Dependence of D·p and D·x on s_f: $D := 0.6$ $s_f := 5, 10 .. 200$



Note that productivity D·p increases linearly with s_f; thus, Profit also increases quite linearly with s_f.
This seems to be true for various models (with substrate inhibition). Solubility limits the maximum s_f.