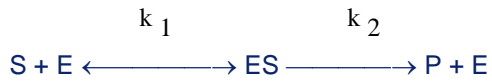


Derive Michaelis-Menten reaction rate expression. Many assumptions lead to the same saturation form.

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Method #1 Equilibrium Assumption.



Given

$$1. \text{ dp/dt=rate=v}$$

$$v = k_2 \cdot ES$$

$$2. \text{ Conservation of enzyme species } E_0 = E + ES$$

$$3. \text{ Equilibrium Assumption for the first step. } k_1 \cdot S \cdot E = k_i \cdot ES$$

Legend

S ... substrate

P ... product

E ... unbound enzyme

ES ... enzyme-substrate complex

(Just set up the equations and let Mathcad do the rest. We have three equations, and we can choose to solve for any of the three unknowns -- E, ES, and v) Find the analytical expression (via Math|SmartMath|).

$$\text{Find}(E, ES, v) \rightarrow \begin{bmatrix} \frac{1}{(k_1 \cdot S + k_i)} \cdot k_i \cdot E_0 \\ E_0 \cdot k_1 \cdot \frac{S}{(k_1 \cdot S + k_i)} \\ k_2 \cdot E_0 \cdot k_1 \cdot \frac{S}{(k_1 \cdot S + k_i)} \end{bmatrix}$$

Thus, the last row is the analytical expression for v.

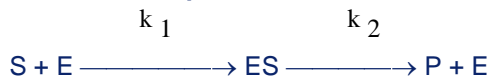
$$k_2 \cdot k_1 \cdot S \cdot \frac{E_0}{(k_1 \cdot S + k_i)}$$

Dividing the numerator and denominator by k_1 gives:

$$v = \frac{k_2 \cdot E_0 \cdot S}{\frac{k_i}{k_1} + S}$$

Thus, by defining: $v_m = k_2 \cdot E_0$ $K_m = \frac{k_i}{k_1}$ we reach the Michaelis-Menten form:

$$v(s) = \frac{v_m \cdot s}{K_m + s}$$

Method #2 Sequential Irreversible Reaction Assumption.

Given

1. $dp/dt = \text{rate} = v \qquad v = k_2 \cdot ES$
2. Conservation of enzyme species $E_0 = E + ES$
3. Irreversible first step without accumulation of intermediate species ES. $k_1 \cdot S \cdot E = k_2 \cdot ES$

$$\text{Find}(E, ES, v) \Rightarrow \begin{bmatrix} \frac{1}{(k_1 \cdot S + k_2)} \cdot k_2 \cdot E_0 \\ E_0 \cdot k_1 \cdot \frac{S}{(k_1 \cdot S + k_2)} \\ k_2 \cdot E_0 \cdot k_1 \cdot \frac{S}{(k_1 \cdot S + k_2)} \end{bmatrix}$$

Thus, by defining: $v_m = k_2 \cdot E_0$ $K_m = \frac{k_2}{k_1}$ we reach the Michaelis-Menten form: $v(s) = \frac{v_m \cdot s}{K_m + s}$

Method #3 Quasi-Steady State Assumption. (Briggs & Haldane)

Given

1. $dp/dt = \text{rate} = v \qquad v = k_2 \cdot ES$
2. Conservation of enzyme species $E_0 = E + ES$
3. Quasi-Steady State Assumption (for the intermediate species ES) $d(ES)/dt = 0$

$$\text{Find}(E, ES, v) \Rightarrow \begin{bmatrix} E_0 \cdot \frac{(k_i + k_2)}{(k_1 \cdot S + k_i + k_2)} \\ E_0 \cdot k_1 \cdot \frac{S}{(k_1 \cdot S + k_i + k_2)} \\ k_2 \cdot E_0 \cdot k_1 \cdot \frac{S}{(k_1 \cdot S + k_i + k_2)} \end{bmatrix} \qquad k_1 \cdot S \cdot E - k_i \cdot ES - k_2 \cdot ES = 0$$

Thus, by defining: $v_m = k_2 \cdot E_0$ $K_m = \frac{k_i + k_2}{k_1}$ we reach the Michaelis-Menten form: $v(s) = \frac{v_m \cdot s}{K_m + s}$

This last form contains the first two assumptions:

For $k_2 \ll k_i$, $K_m \rightarrow k_i/k_1$ (which is Assumption #1)

For $k_2 \gg k_i$, $K_m \rightarrow k_2/k_1$ (which is Assumption #2)