Optimum dilution rate for cell productivity with substrate inhibition.
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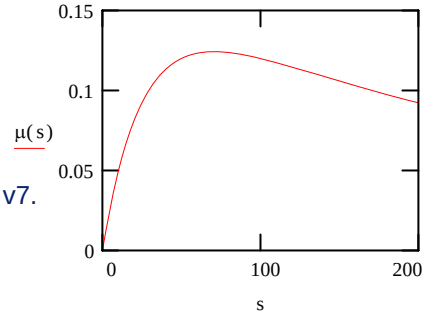
Operating condition:

$s_f := 200$

Model parameters:

$\mu_m := 0.3$ $K := 50$ $K_i := 0.01$ $A := 0.004$ $B := 0.001$

$\mu(s) := \frac{\mu_m \cdot s}{K + s + K_i \cdot s^2}$ $Y(s) := A + B \cdot s$ $s := 0 .. s_f$



Maximum specific growth rate μ: $s := 10$ ... initial guess

Given $\frac{d}{ds}\mu(s) = 0$ $s := \text{Find}(s)$ $s = 70.711$

$\mu(s) = 0.124$

An initial guess of s=0 is o.k. in Mathcad v5, but no good in v7.

Note that the "root" function does not seem to be as accurate:

$s := 0$ $s := \text{root}\left(\frac{d}{ds}\mu(s), s\right)$ $s = 51.301$ $\mu(s) = 0.121$

Initial guesses:

$x := 1$ $s := 0$

Given Steady-state equations:

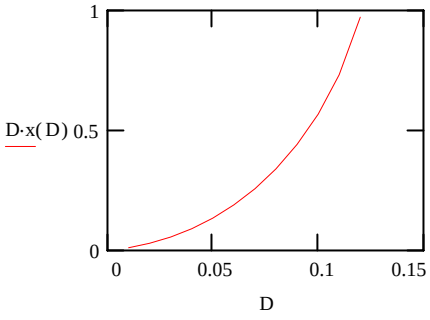
dx/dt: $0 = (\mu(s) - D) \cdot x$

ds/dt: $0 = D \cdot (s_f - s) - \frac{1}{Y(s)} \cdot \mu(s) \cdot x$

$\text{ans}(D) := \text{Find}(x, s)$ $x(D) := \text{ans}(D)_0$

An example: $\text{ans}(0.1) = \begin{pmatrix} 5.683 \\ 29.289 \end{pmatrix}$

Dependence of D*x on D: $D := 0.01, 0.02 .. 0.124$



Note that cell productivity increases monotonically with D, right up to maximum μ, at which point cell washout occurs. The maximum cell productivity occurs at: $D := 0.124$

The maximum cell productivity is: $D \cdot x(D) = 1.155$