Immobilized Enzyme on a (CSTR) Reactor Surface with Product Inhibition (/w physical units followed by dimensionless analysis.)
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Operating parameters:

$F := 3 \cdot cm^3 \cdot sec^{-1}$ ... volumetric flow rate

$A := 100 \cdot cm^2$ ... surface area of immobilized enzyme

$s_f := 10 \cdot gm \cdot liter^{-1}$ ... feed substrate concentration

$k_L := 0.01 \cdot cm \cdot sec^{-1}$ ... mass transfer coefficient

Reaction rate parameters:

$v_m := 0.0001 \cdot gm \cdot sec^{-1} \cdot cm^{-2}$ ... maximum reaction rate

$K_m := 1 \cdot gm \cdot liter^{-1}$ ... Michaelis-Menten constant

$K_p := 1$ ... product inhibition constant

$$v(s,p) := \frac{v_m \cdot s}{K_m + s + K_p \cdot p}$$

Material balance equations for the rate of conversion are given below:

$$r_1(s_b) := F \cdot (s_f - s_b)$$

$$r_2(p_b) := F \cdot p_b$$

$$r_3(s_b, s) := A \cdot k_L \cdot (s_b - s)$$

$$r_4(p_b, p) := A \cdot k_L \cdot (p - p_b)$$

$$r_5(s,p) := A \cdot v(s,p)$$

The above 5 equations can be reduced to the following equation:

$$\frac{F}{v_m \cdot A} \cdot \frac{s_f - s}{1 + \frac{F}{k_L \cdot A}} = \frac{s}{K_m + K_p \cdot s_f + (1 - K_p) \cdot s}$$

Non-dimensionalize the last equation with

$$x := \frac{s}{s_f} \qquad K := \frac{K_m}{s_f} \qquad \beta := \frac{F \cdot s_f}{v_m \cdot A} \qquad \Gamma := \frac{F}{k_L \cdot A} \qquad \alpha := \frac{\beta}{1 + \Gamma}$$

$\beta = 3$ $\Gamma = 3$ $\alpha = 0.75$

Thus, the non-dimensional equation to be solved is:

$$f(x, \alpha, K, K_p) := \alpha \cdot (1 - x) - \frac{x}{K + K_p + (1 - K_p) \cdot x} = 0$$

Sample calculation based on the given parameters:

$x := 1$ (provide initial guess)

$x := \mathrm{root}\left(f\left(x, \alpha, K, K_p\right), x\right)$ $x = 0.452$

$s := x \cdot s_f$ $s = 4.521 \cdot gm \cdot liter^{-1}$ ... Substrate conc. at the reaction surface

$p := s_f - s$ $p = 5.479 \cdot gm \cdot liter^{-1}$ ... Product conc. at the reaction surface

$s_b := \dfrac{\Gamma \cdot s_f + s}{1 + \Gamma}$ $s_b = 8.63 \cdot gm \cdot liter^{-1}$ ... Substrate conc. in the bulk phase or reactor exit

$p_b := \dfrac{p}{1 + \Gamma}$ $p_b = 1.37 \cdot gm \cdot liter^{-1}$ ... Product conc. in the bulk phase or reactor exit

Check (All rates should be identical):

$r_1\left(s_b\right) = 0.00411 \cdot gm \cdot sec^{-1}$

$r_2\left(p_b\right) = 0.00411 \cdot gm \cdot sec^{-1}$

$r_3\left(s_b, s\right) = 0.00411 \cdot gm \cdot sec^{-1}$

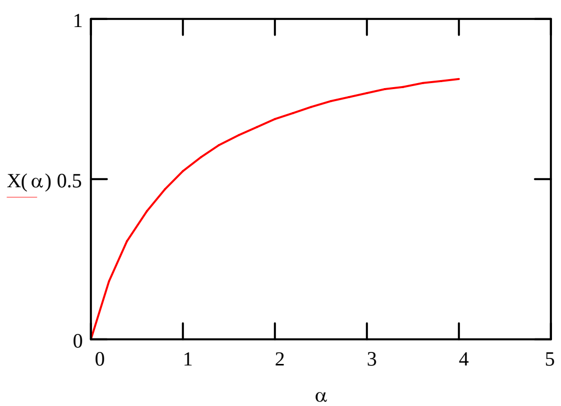
$r_4\left(p_b, p\right) = 0.00411 \cdot gm \cdot sec^{-1}$

$r_5(s, p) = 0.00411 \cdot gm \cdot sec^{-1}$

Since α contains all the operating parameters, the effect of various operating conditions on conversion can be studied by varying α.

$X(\alpha) := \mathrm{root}\left(f\left(x, \alpha, K, K_p\right), x\right)$

$\alpha := 0, 0.2 .. 4$



α	$X(\alpha)$
0	0
0.2	0.18
0.4	0.306
0.6	0.398
0.8	0.468
1	0.524
1.2	0.569
1.4	0.606
1.6	0.638
1.8	0.664
2	0.687
2.2	0.708
2.4	0.725
2.6	0.741
2.8	0.755
3	0.767
3.2	0.779
3.4	0.789
3.6	0.798
3.8	0.807

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4	0.815