Dynamic simulation of microbial growth -- with **cell recycle**
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Operating conditions:

$s_f := 10$ ... Feed substrate concentration

$D := 0.55$ ... Dilution rate

Constitutive relations:

$\mu_m := 0.5$ ... maximum specific growth rate

$K := 1$ ... Monod constant

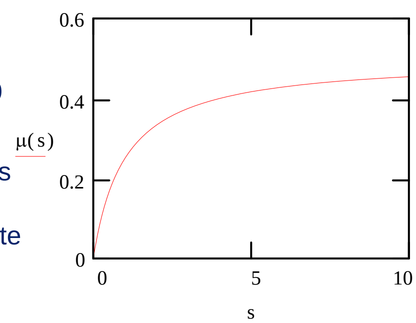
$Y := 0.5$ ... yield coefficient

$\mu(s) := \frac{\mu_m \cdot s}{K + s}$ ... Monod specific growth rate

$\alpha := 1$ ... recycle ratio

$C := 1.5$ ... concentration factor, with 1+α*(1-C)>0

$s := 0, 0.1 .. s_f$



Dynamic Equations:

$dxdt(x,s) := (\mu(s) - (1 + \alpha - \alpha \cdot C) \cdot D) \cdot x$ $x_0 := 1$ ... biomass

$dsdt(x,s) := D \cdot (s_f - s) - \frac{\mu(s) \cdot x}{Y}$ $s_0 := s_f$ ... substrate

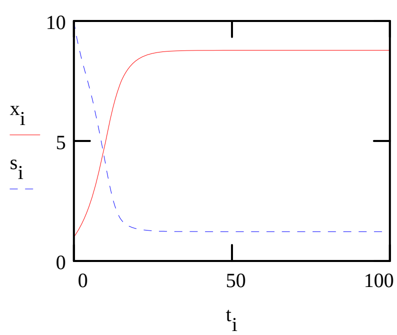
Combine individual functions into a vector function.

$ydot(t,y) := \begin{pmatrix} dxdt(y_0, y_1) \\ dsdt(y_0, y_1) \end{pmatrix}$ I.C.: $y_{initial} := \begin{pmatrix} x_0 \\ s_0 \end{pmatrix}$

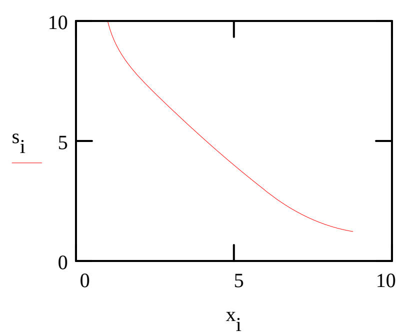
Integrate both ODEs from $t_0 := 0$ to $t_f := 100$ in $nstep := 200$

$yout := rkfixed(y_{initial}, t_0, t_f, nstep, ydot)$ $t := yout^{\langle 0 \rangle}$ $x := yout^{\langle 1 \rangle}$ $s := yout^{\langle 2 \rangle}$ $i := 0 .. last(t)$

Plot of state variables



Phase diagram



Cell recycle moderates the bioreactor dynamics by adding an extra dynamic layer and allows the bioreactor to be operated at a higher dilution rate before cell washout occurs.

Cell Recycle with Time-Delay in the Recycle Line

$i := 0 .. nstep \qquad h := \dfrac{t_f - t_0}{nstep}$ ... step size $\qquad t_i := i \cdot h$

Steps of delay $\quad delay := 4 \qquad$ Actual time-delay: $\quad delay \cdot h = 2$

Initial conditions (up to the time-delay)

$i := 0 .. delay \qquad x_i := x_0 \qquad s_i := s_0$

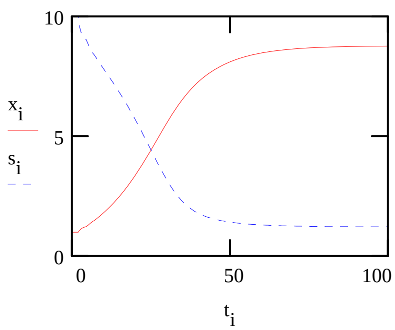
Integration by Euler's Method

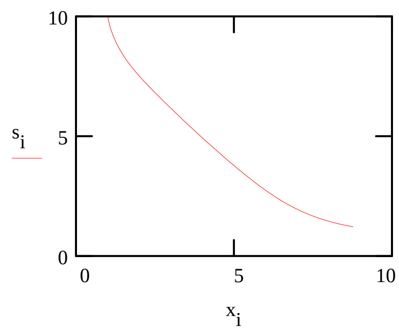
$i := delay .. nstep$

$$\begin{pmatrix} x_{i+1} \\ s_{i+1} \end{pmatrix} := \begin{bmatrix} x_i + \left[-(1+\alpha) \cdot D \cdot x_i + \alpha \cdot C \cdot D \cdot x_{i-delay} + \mu(s_i) \cdot x_i \right] \cdot h \\ s_i + \left[D \cdot s_f - (1+\alpha) \cdot D \cdot s_i + \alpha \cdot D \cdot s_{i-delay} - \dfrac{\mu(s_i) \cdot x_i}{Y} \right] \cdot h \end{bmatrix}$$

Plot

$i := 0 .. nstep$





Time-delay in the recycle loop is manifested in the more sluggish response.