

Fed-Batch Reactor with Product Formation (Repeated Cyclic (Fill-Harvest) Operation)
Instructor: Nam Sun Wang

Operation conditions:

- $s_f := 1$... Feed substrate concentration (g/L)
 $F := 0.2$... Flow rate (L/h) ← **Change this number to optimize productivity**
 $V := 1$... Fermentor volume (L)

Constitutive relations:

- $\mu(s) := \Phi(s) \cdot \frac{0.7 \cdot s}{0.02 + s}$... Monod specific growth rate The following equation avoids overshoot:
 $Y(s) := 0.5$... substrate-cell yield coefficient $\mu(s) := \text{if}(s \leq 0, 0, \frac{\mu_m \cdot s}{K + s})$
 $Y_p := 0.15$... substrate-product yield coefficient
 $\alpha(s) := 0.1$... growth related product formation
 $\beta(s) := \Phi(s) \cdot 0.02$... maintenance-related product formation

Dynamic Equations:

$$\begin{aligned}
 dxdt(x, s, p, v) &:= \left(\mu(s) - \frac{F}{v} \right) \cdot x \\
 dsdt(x, s, p, v) &:= \frac{F}{v} \cdot (s_f - s) - \frac{1}{Y(s)} \cdot \mu(s) \cdot x - \frac{1}{Y_p} \cdot (\alpha(s) \cdot \mu(s) \cdot x + \beta(s) \cdot x) \\
 dpdt(x, s, p, v) &:= \alpha(s) \cdot \mu(s) \cdot x + \beta(s) \cdot x - \frac{F}{v} \cdot p \\
 dvd(t, x, s, p, v) &:= F \\
 ydot(t, y) &:= \begin{bmatrix} dxdt(y_0, y_1, y_2, y_3) \\ dsdt(y_0, y_1, y_2, y_3) \\ dpdt(y_0, y_1, y_2, y_3) \\ dvd(y_0, y_1, y_2, y_3) \end{bmatrix}
 \end{aligned}$$

Iteratively update the initial conditions of x, s, and p to match the final conditions.

Initial conditions: $x_0 := 0.342$ $s_0 := 0.00663$ $p_0 := 0.0463$ $v_0 := 0.05 \cdot V$ (5% inoculum)

$$y_{\text{initial}} := \begin{bmatrix} x_0 \\ s_0 \\ p_0 \\ v_0 \end{bmatrix} \begin{array}{l} \dots \text{ biomass} \\ \dots \text{ substrate} \\ \dots \text{ product} \\ \dots \text{ volume} \end{array}$$

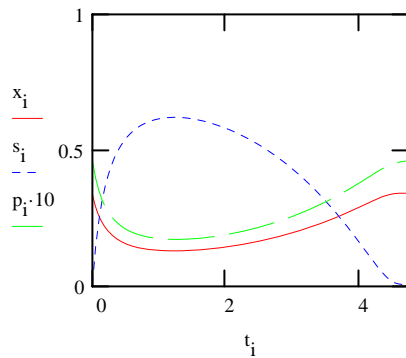
Solve both sets of ODEs (integrate from $t_0 := 0$ to $t_f := \frac{V - v_0}{F}$ in $N := 100$ steps

$$y_{\text{out}} := \text{rkfixed}(y_{\text{initial}}, t_0, t_f, N, ydot)$$

$$t := y_{\text{out}}^{<0>} \quad x := y_{\text{out}}^{<1>} \quad s := y_{\text{out}}^{<2>} \quad p := y_{\text{out}}^{<3>} \quad v := y_{\text{out}}^{<4>}$$

$$x_N = 0.342 \quad s_N = 0.00663 \quad p_N = 0.0463 \quad v_N = 1$$

Plots of state variables $i := 0 \dots \text{last}(t)$



Productivity

$$\text{Cells ... } \frac{v_N \cdot x_N - v_0 \cdot x_0}{t_f} = 0.06847$$

$$\text{Product ... } \frac{v_N \cdot p_N - v_0 \cdot p_0}{t_f} = 0.00926$$

Change F at the beginning of this document and see how productivity of cells and products change. The following table was manually constructed by following this what-if procedure for each flow rate.

$j := 0 \dots 100$

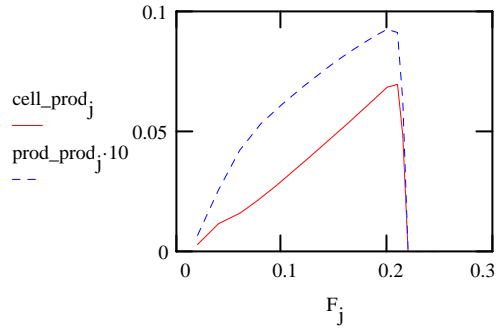
$F_j :=$ $xN_j :=$ $sN_j :=$ $pN_j :=$ $\text{cell_prod}_j :=$ $\text{prod_prod}_j :=$

0.02	0.290	0.0000	0.0665	0.00295	0.00067
0.04	0.290	0.0000	0.0643	0.01161	0.00257
0.06	0.265	0.00176	0.0701	0.01593	0.00421
0.08	0.277	0.00182	0.0665	0.02218	0.00532
0.10	0.295	0.00249	0.0612	0.02946	0.00612
0.12	0.308	0.00320	0.0572	0.03693	0.00686
0.14	0.318	0.00394	0.0539	0.04456	0.00755
0.15	0.323	0.00433	0.0525	0.04844	0.00787
0.16	0.327	0.00472	0.0511	0.05236	0.00818
0.17	0.331	0.00513	0.0498	0.05632	0.00847
0.18	0.335	0.00554	0.0486	0.06033	0.00875
0.19	0.339	0.00597	0.0475	0.06438	0.00902
0.20	0.342	0.00663	0.0463	0.06847	0.00926
0.21	0.332	0.04637	0.0435	0.06969	0.00913
0.215	0.225	0.35485	0.0292	0.04843	0.00628
0.22	0.	1	0	0	0
0.25	0.	1	0	0	0

← optimum flow rate

← washout above F=0.215

$j := 0 \dots \text{last}(F)$



From the table and the plot, maximum productivity of cells and product occur at $F=0.21$ and $F=0.20$, respectively. These numbers depend on the operating conditions. The general shape of the productivity-versus-flow rate curve is similar to that of noncyclic operation. In particular, there is a very sharp decrease in the productivity/profit when flow rate increases past the optimum value. It is safer to operate at a flow rate slightly below the optimum value to avoid washout. Note that at maximum productivity, the substrate is just about depleted when the run is terminated.