Fed-Batch Reactor with Product Formation (two flow rates)

Instructor: Nam Sun Wang

Flow rates (L/h); switch from F_1 to F_2 at $t=t_1$

$F_1 := 0 \quad F_2 := 0.2 \quad t_1 := 2.5$

$F(t) := \text{if}(t<t_1, F_1, F_2)$

Other Operation conditions:

$s_f := 1$ ... Feed substrate concentration (g/L)

$V := 1$ ... Fermentor volume (L)

Constitutive relations:

$\mu(s) := \Phi(s)\cdot\frac{0.7\cdot s}{0.02+s}$ ... Monod specific growth rate

The following equation avoids overshoot: $\mu(s) := \text{if}\left(s\le 0, 0, \frac{\mu_m\cdot s}{K+s}\right)$

$Y(s) := 0.5$ ... substrate-cell yield coefficient

$Yp := 0.15$ ... substrate-product yield coefficient

$\alpha(s) := 0.1$ ... growth related product formation

$\beta(s) := \Phi(s)\cdot 0.02$ ... maintenance-related product formation

Dynamic Equations:

$$dxdt(t,x,s,p,v) := \left(\mu(s) - \frac{F(t)}{v}\right)\cdot x$$

$$dsdt(t,x,s,p,v) := \frac{F(t)}{v}\cdot(s_f - s) - \frac{\mu(s)}{Y(s)}\cdot x - \frac{1}{Yp}\cdot(\alpha(s)\cdot\mu(s)\cdot x + \beta(s)\cdot x)$$

$$dpdt(t,x,s,p,v) := \alpha(s)\cdot\mu(s)\cdot x + \beta(s)\cdot x - \frac{F(t)}{v}\cdot p$$

$$dvdt(t,x,s,p,v) := F(t)$$

$$ydot(t,y) := \begin{bmatrix} dxdt(t,y_0,y_1,y_2,y_3) \\ dsdt(t,y_0,y_1,y_2,y_3) \\ dpdt(t,y_0,y_1,y_2,y_3) \\ dvdt(t,y_0,y_1,y_2,y_3) \end{bmatrix}$$

Initial conditions: $x_0 := 0.1 \quad s_0 := s_f \quad p_0 := 0 \quad v_0 := 0.1$

$$y_{initial} := \begin{bmatrix} x_0 \\ s_0 \\ p_0 \\ v_0 \end{bmatrix}$$

... biomass

... substrate

... product

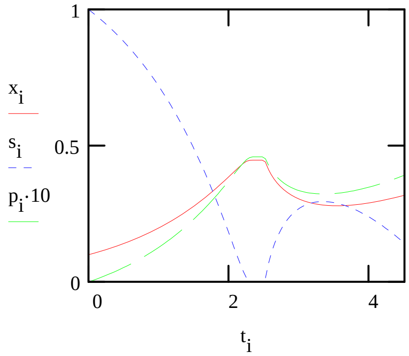
... volume

Solve both sets of ODEs (solve from $t_0 := 0$ to $t_f := \frac{V - v_0 - F_1\cdot t_1}{F_2}$ in $nstep := 100$

$yout := \text{rkfixed}(y_{initial}, t_0, t_f, nstep, ydot)$

$t := yout^{\langle 0\rangle} \quad x := yout^{\langle 1\rangle} \quad s := yout^{\langle 2\rangle} \quad p := yout^{\langle 3\rangle} \quad v := yout^{\langle 4\rangle}$

Plots of state variables $i := 0 .. \mathrm{last}(t)$



Productivity

Cells ... $\dfrac{v_{nstep} \cdot x_{nstep} - v_0 \cdot x_0}{t_f} = 0.03296$

Product ... $\dfrac{v_{nstep} \cdot p_{nstep} - v_0 \cdot p_0}{t_f} = 0.00433$