Fed-Batch Reactor with Product Formation (two flow rates)
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Flow rates (L/h); switch from F_1 to F_2 at t=t_1, i.e., midway through the fermentation.

$F_1 := 0.35 \quad F_2 := 0$

$v_0 := 0.1 \quad V := 1$

$$t_1 := \frac{V - v_0}{F_1 + F_2} \qquad F(t) := \text{if}\left(t < t_1, F_1, F_2\right)$$

Other Operation conditions:

$s_f := 1$ ... Feed substrate concentration (g/L)

Constitutive relations:

$$\mu(s) := \Phi(s) \cdot \frac{0.7 \cdot s}{0.02 + s}$$ ... Monod specific growth rate

The following equation avoids overshoot:

$$\mu(s) := \text{if}\left(s \le 0, 0, \frac{\mu_m \cdot s}{K + s}\right)$$

$Y(s) := 0.5$ ... substrate-cell yield coefficient

$Yp := 0.15$ ... substrate-product yield coefficient

$\alpha(s) := 0.1$ ... growth related product formation

$\beta(s) := \Phi(s) \cdot 0.02$ ... maintenance-related product formation

Dynamic Equations:

$$dxdt(t, x, s, p, v) := \left(\mu(s) - \frac{F(t)}{v}\right) \cdot x$$

$$dsdt(t, x, s, p, v) := \frac{F(t)}{v} \cdot \left(s_f - s\right) - \frac{\mu(s)}{Y(s)} \cdot x - \frac{1}{Yp} \cdot (\alpha(s) \cdot \mu(s) \cdot x + \beta(s) \cdot x)$$

$$dpdt(t, x, s, p, v) := \alpha(s) \cdot \mu(s) \cdot x + \beta(s) \cdot x - \frac{F(t)}{v} \cdot p$$

$$dvdt(t, x, s, p, v) := F(t)$$

$$ydot(t, y) := \begin{bmatrix} dxdt\left(t, y_0, y_1, y_2, y_3\right) \\ dsdt\left(t, y_0, y_1, y_2, y_3\right) \\ dpdt\left(t, y_0, y_1, y_2, y_3\right) \\ dvdt\left(t, y_0, y_1, y_2, y_3\right) \end{bmatrix}$$

Initial conditions: $x_0 := 0.1 \quad s_0 := s_f \quad p_0 := 0$

$$y_{initial} := \begin{bmatrix} x_0 \\ s_0 \\ p_0 \\ v_0 \end{bmatrix}$$

... biomass

... substrate

... product

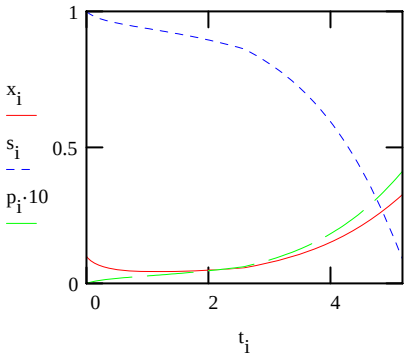
... volume

Solve both sets of ODEs (solve from $t_0 := 0$ to $t_f := 2 \cdot t_1$ in $nstep := 100$

$yout := rkfixed\left(y_{initial}, t_0, t_f, nstep, ydot\right)$

$t := yout^{\langle 0 \rangle} \quad x := yout^{\langle 1 \rangle} \quad s := yout^{\langle 2 \rangle} \quad p := yout^{\langle 3 \rangle} \quad v := yout^{\langle 4 \rangle}$

Plots of state variables $i := 0 .. last(t)$



Productivity

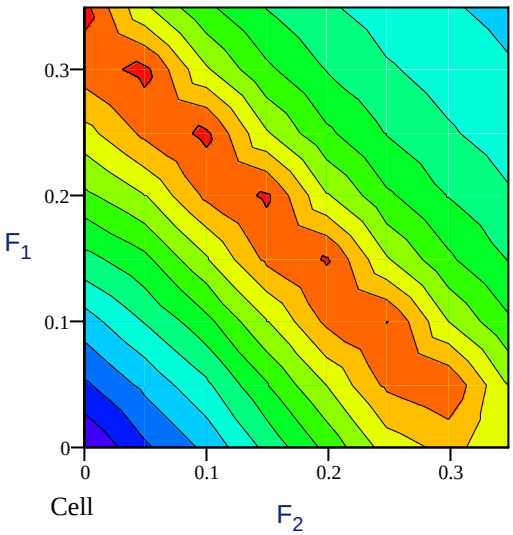
Cells ... $\frac{v_{nstep} \cdot x_{nstep} - v_0 \cdot x_0}{t_f} = 0.06147$

Product ... $\frac{v_{nstep} \cdot p_{nstep} - v_0 \cdot p_0}{t_f} = 0.008$

Change F1 & F2 at the beginning of this document and see how productivity of cells and products change. The following table was entered manually by following this what-if procedure.

Cell :=

F_1 \ F_2	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35
0.00	0.00000	0.00918	0.01773	0.02829	0.03870	0.04812	0.05772	0.06147
0.05	0.00908	0.01616	0.02537	0.03374	0.04477	0.05628	0.06130	0.04571
0.10	0.01617	0.02398	0.03348	0.04434	0.05599	0.06111	0.04566	0.03602
0.15	0.02626	0.03446	0.04468	0.05601	0.06086	0.04559	0.03599	0.02985
0.20	0.03641	0.04520	0.05603	0.06053	0.04550	0.03596	0.02983	0.02567
0.25	0.04714	0.05609	0.06005	0.04538	0.03591	0.02981	0.02565	0.022
0.30	0.05160	0.05916	0.04519	0.03585	0.02979	0.02564	0.02267	0.020
0.35	0.04623	0.04483	0.03576	0.02975	0.02562	0.02266	0.02046	0.018



F_1

0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	
0.00000	0.00151	0.00313	0.00404	0.00514	0.00634	0.00762	0.00800	0.00
0.00154	0.00352	0.00490	0.00656	0.00735	0.00805	0.00798	0.00592	0.05
0.00331	0.00529	0.00659	0.00748	0.00813	0.00795	0.00591	0.00466	0.10
0.00461	0.00623	0.00736	0.00810	0.00792	0.00590	0.00465	0.00386	0.15
0.00559	0.00717	0.00806	0.00788	0.00589	0.00465	0.00386	0.00332	0.20
0.00656	0.00799	0.00783	0.00588	0.00465	0.00385	0.00332	0.002	0.25
0.00675	0.00772	0.00586	0.00464	0.00385	0.00331	0.00293	0.002	0.30
0.00603	0.00581	0.00463	0.00385	0.00331	0.00293	0.00264	0.002	0.35

Product :=

F_2