

Demonstrate symbolic solution of Michaelis-Menten enzyme kinetics expression in a batch reactor
Instructor: Nam Sun Wang

The Michaelis-Menten rate expression is given by:

$$v = \frac{d}{dt} s = \frac{v_m \cdot s}{K_m + s}$$

Rearranging the above equations:

$$\frac{K_m + s}{s} \cdot ds = v_m \cdot dt$$

LHS (integrate from S_0 to S): RHS (integrate from 0 to t):

$$\int_{S_0}^S \frac{K_m + s}{s} ds = \int_0^t v_m dT$$

After marking the entire equation, the above LHS and RHS can be integrated by MathCAD via |Symbolic|Evaluate|Evaluate Symbolically| to yield:

$$S + K_m \cdot \ln(S) - S_0 - K_m \cdot \ln(S_0) = v_m \cdot t$$

After marking K_m and simplify via |Symbolic|Collect on Subexpression| to yield:

$$(\ln(S) - \ln(S_0)) \cdot K_m + S - S_0 = v_m \cdot t \quad \dots \text{Analytical solution (eqn 3.16 of Shuler \& Kargi)}$$

For example, the substrate concentration at a specified time (e.g., $t=1$) can be calculated with one of the algebraic equation solvers:

$$K_m := 1 \quad v_m := 1 \quad S_0 := 10 \quad \dots \text{Provide numerical values for constants}$$

$$S := 1 \quad \dots \text{initial guess}$$

Given

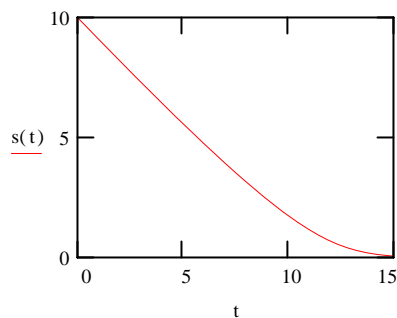
$$(\ln(S) - \ln(S_0)) \cdot K_m + S - S_0 = v_m \cdot t$$

$$s(t) := \text{Find}(S) \quad \dots \text{substrate conc. as a function of } t \text{ is the solution to the above equation.}$$

$$s(0) = 10 \quad \dots \text{check, o.k.; at time 0, the substrate conc. is } S_0$$

$$s(1) = 9.095$$

Plot the time course: $t := 0, 0.1 .. 15$



Note that for the given set of kinetic parameters, the rate is almost linear for the most part of the reaction.