Surveying and trigonometry

Materials:

- protractor
- 2m string or thread
- 2 small heavy objects (for plumb bobs)
- drinking straw
- paper clip
- tape measure
- pushpins
- tape (any kind)
- broom handle (available for about Kw 2000 at Game or Shoprite)
- 6 bricks or stones
- calculator with tangent function

Suitable for: 3-30 students

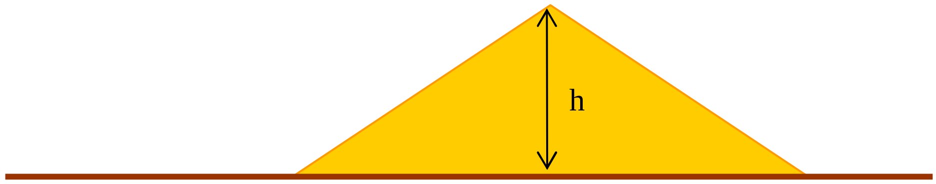
Time required: 1.5 hours

Topics discussed: trigonometry, changing the subject of an equation

I. Introduction

Tell the students that we will show one situation in which trigonometry can be extremely useful. We will use the tangent function to calculate the height of an object, an object whose height could in principle be impossible to measure directly. This technique is similar to methods which are used in surveying.

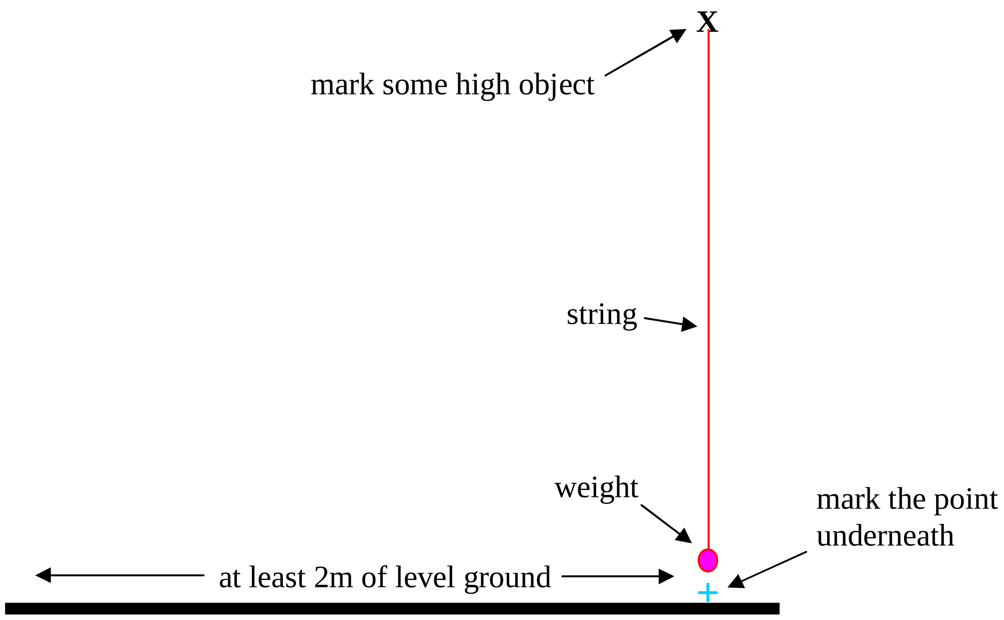
Draw the following figure:



Suppose an explorer came upon a giant pyramid, like those in Egypt, and wanted to record the height. There would be no way to measure this quantity directly, by putting a ruler down through the middle of the pyramid from the peak to the base! A surveyor would have the same problem if he/she wanted to know the height of a mountain. Today we will develop a method which can find the height in these situations, but it will require us to use trigonometry.

II. Location for activity

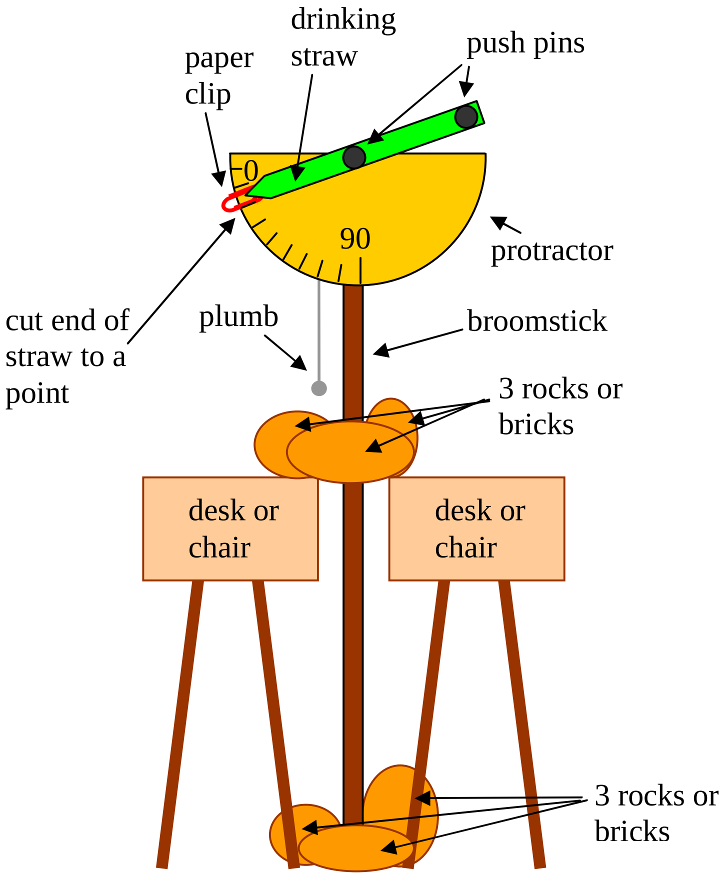
Before running the activity, you will need to select a proper location. You will need to find a stretch of at least 2 metres of **level** pavement, floor, or walkway. It is crucial that this area not have a significant slant. You will also need to select an object whose height will be measured: you could use a light fixture, a window ledge, etc. Mark the object with an “X” somehow, perhaps by taping a small piece of paper to it:



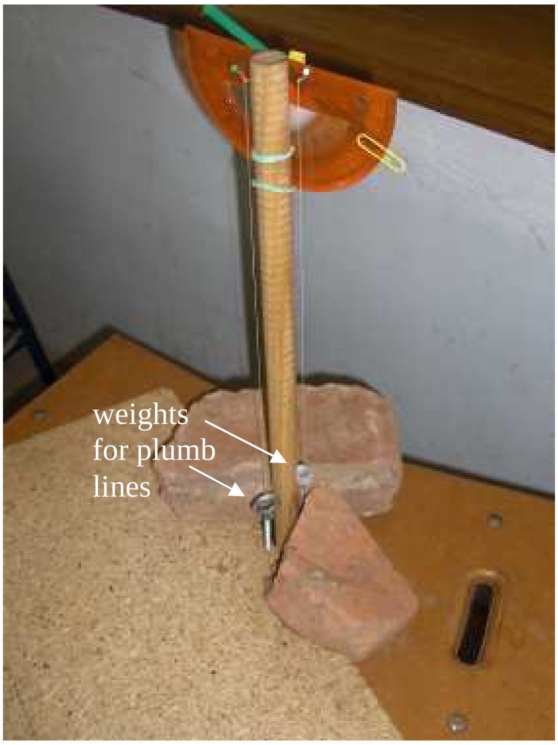
Next, before the students arrive, we will need to mark the point on the ground which is directly underneath the X. This is done by hanging a “plumb line,” which consists of a string with any weight attached. The length of the string should be almost long enough that the weight touches the ground, but just about 1cm too short. Once the weight stops swinging back and forth, then mark the point underneath with a “+” sign made of tape. Now remove the plumb line: it should not be attached to the X when the pupils arrive.

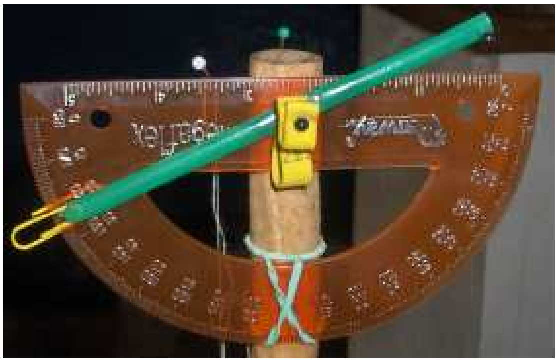
III. Clinometer

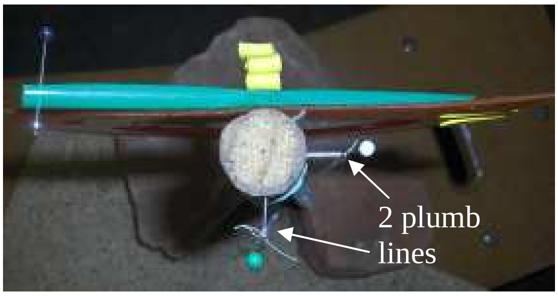
Before the lesson, construct a “clinometer” like the one shown here:

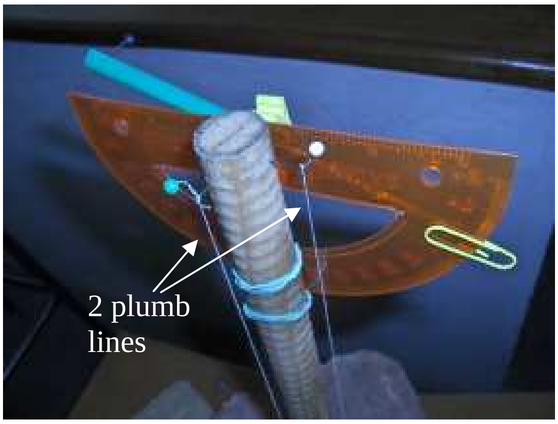


Here are some photos of the clinometer we actually used for this activity at two schools in January 2009:

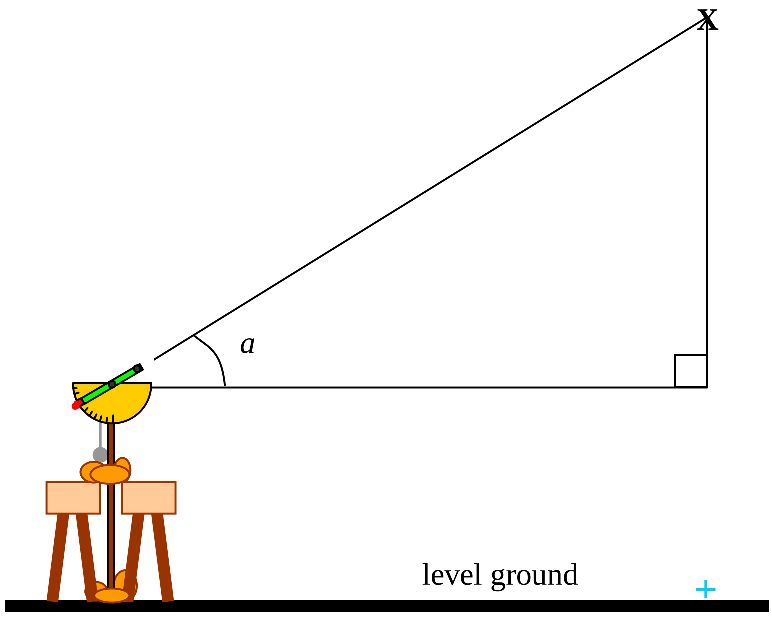








The purpose of the clinometer is to measure the angle *a* in the figure below. You can think of *a* as the angle between an imaginary horizontal line and the line-of-sight to the point “X”.



To measure *a*, the angle of the straw is adjusted until you can see the X

The protractor must be attached to the broomstick somehow - in the photos you can see how I have used the green rubber band for this purpose; tape could be used instead.

Cut one end of the straw - the end which is against the round edge of the protractor - to a point. This is done to make it easier to read the straw's angle on the protractor scale. The paper clip is used to hold the straw up against the scale, since otherwise the small gap between the straw and scale makes the measurement ambiguous.

The purpose of the two triplets of bricks is to secure the broomstick at a certain angle, while still allowing the stick to be rotated about its axis. We want the broomstick to be perfectly up-and-down; the angle of the broomstick can be adjusted by moving the bottom three bricks while leaving the top three alone, or by moving the top three bricks while leaving the bottom three alone. In the photographs above, the wooden board has been used in place of one of the bricks.

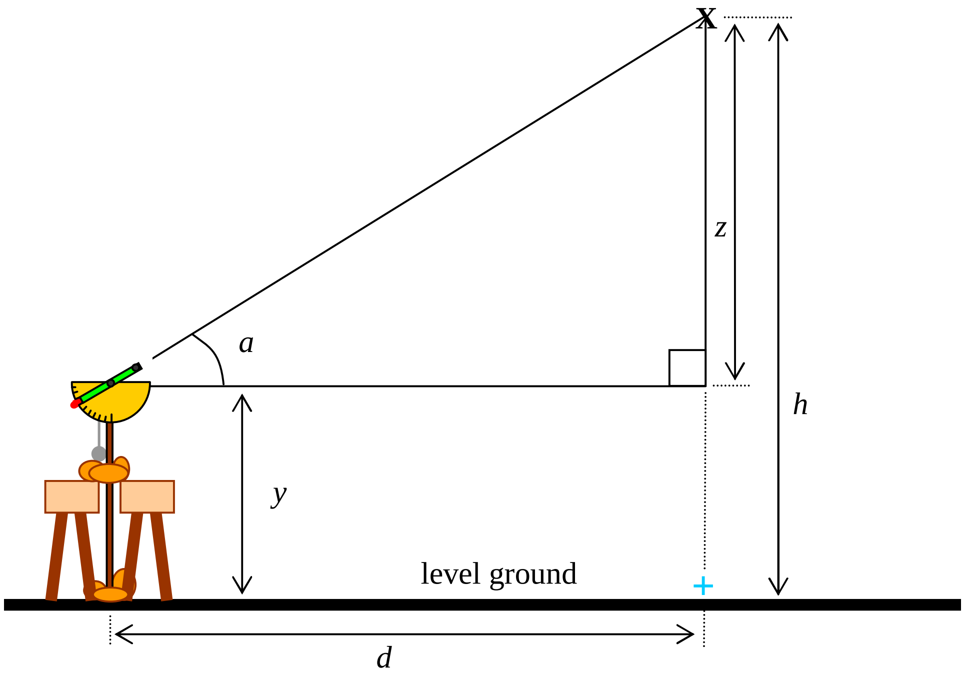
To be certain when the broomstick is perfectly up-and-down, we use a pair of "plumb lines" hung from its top. Each plumb consists of a thread or string with a weight attached to the end. The two lines are hung at 90 degrees from each other around the broomstick if you look down at the broomstick from above. Gravity will pull the weights directly downward. You should adjust the angle of the broomstick by moving the 6 bricks such that the broomstick is aligned to the plumb lines. There are two plumb lines so that you can align both the left-right angle and the forward-backward angle. In the photographs above I have used the green and white pushpins to hang the plumb lines about 1cm away from broomstick; this offset allows the weights to hang freely without bumping against the broomstick.

There are two pins through the straw. One pin goes through the centre of the protractor and allows the straw to rotate about this point. The second pin is used for alignment: to measure the angle *a*, look through the straw and rotate it about the first pin until both pins are lined up with the X. You will also need to rotate the broomstick about its axis (i.e. side-to-side).

IV. Deriving a formula for the height

After giving the introduction (section I) to the students, show them where you have placed the "X", so they understand what we are aiming to measure.

Return to a classroom, and draw the following figure on the chalkboard:



Explain that we eventually want to calculate *h* without measuring it directly: we will only measure *a*, *d*, and *y*.

First, ask the students: "What is the relationship between *a*, *d*, and *z*?" Answer: tan(*a*)=*z*/*d*.

Next, ask the students: "We will measure *a* and *d*. How would we calculate *z* from this information?" Answer: change the subject of the formula to get *z*=*d* tan(*a*).

Ask: "Once we know *z* and *y*, how would be calculate *h*?" Answer: *h*=*z*+*y*.

Finally, ask the students to put the pieces together: "So if we measure *a*, *d*, and *y*, what formula would be use to compute *h*?" Answer: *h*=*y*+*d* tan(*a*).

V. Experiment

Now take the students out to the stretch of level ground and measure *a, d,* and *y* using the clinometer or tape measure respectively. To measure *d*, you can measure the distance between the base of the broomstick and the "+" sign you marked with tape before the students arrived. You may wish to give several students a chance to read off the measurements so they all get practice reading the scales; you can average their readings.

Here are some photos of students carrying out the measurements in January 2009:

Return to the classroom, and have each student compute *h* using the formula $h=y+d \tan(a)$.

Now, return to the stretch of level ground and use a tape measure to directly measure *h*.

VI. Discussion

The trigonometric method will probably give a slightly different answer for the height than the direct measurement. Ask the students: "What factors are responsible for the difference?" (possible answers: difficulty in aligning the straw or aligning the clinometer to the plumb lines, wind blowing the plumb lines, the protractor is not exactly straight relative to the broom stick, etc.)

An astute student might make the point that in the case of measuring a real mountain, there would be no way to know the baseline distance *d*. If this point comes up, you can mention that this problem can be circumvented using a method in which a clinometer reading is taken at *two* locations; then the height can be calculated without knowing *d*.

Finally, ask for 1-3 volunteers from the class to explain in their own words: "What was the point of this activity? What did you learn?"