

Sign conventions in VMEC

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This note is based on some preliminary numerical experiments, and I have not yet been able to confirm them by examination of the source code, so there could well be errors.

The following points apply to the case `lrfp=.false.`; I am not sure if anything is different in the `lrfp=.true.` case.

1. The toroidal angle, variously called ϕ , ζ , or v , always increases in a counter-clockwise direction when the plasma is viewed from above. Hence, the cylindrical coordinate system (R, ζ, Z) is right-handed. (I've verified this is true for MAKEGRID, and hence true for free-boundary VMEC, but I'm not sure it needs to be true for fixed-boundary VMEC.)
2. The direction of the poloidal angle, variously called θ or u , always increases as you move from the outboard to inboard side over the top of the plasma. If the boundary shape specified in the input file implies the opposite direction for θ , vmec will flip the sign of some of the shape coefficients to flip the sign of θ . As a result, the Jacobian of the (s, θ, ζ) coordinates

$$\sqrt{g} = \frac{\partial \mathbf{r}}{\partial s} \cdot \frac{\partial \mathbf{r}}{\partial \theta} \times \frac{\partial \mathbf{r}}{\partial \zeta} = \frac{1}{\nabla s \cdot \nabla \theta \times \nabla \zeta} \quad (1)$$

is always negative. Here, \mathbf{r} is the position vector, and s is the toroidal flux normalized to its value at the edge, so $s = 0$ on the magnetic axis and $s = 1$ at the outermost surface. This sign of \sqrt{g} is reflected in the output variable `signgs`, which is always -1 . (Then why is `signgs` saved, if it is always the same?)

3. These previous points imply that the rotational transform $\iota = d(\theta + \lambda)/d\zeta$ is positive if the magnetic field lines spiral with a left-handed orientation, whereas $\iota < 0$ if the field lines make a right-handed spiral.
4. A positive value for the input toroidal flux variable `phiedge` corresponds to the magnetic field pointing in the $\nabla \zeta$ direction, (counter-clockwise when viewed from above.) This flux satisfies

$$\text{phiedge} = \int_0^s ds' \int_0^{2\pi} d\theta |\sqrt{g}| \mathbf{B} \cdot \nabla \zeta, \quad (2)$$

where the absolute value must be included around the $|\sqrt{g}|$ factor.

5. The output toroidal flux quantities `phi` and `phipf` have the same sign as `phiedge`. However, the output toroidal flux quantity `phips` differs in sign by `signgs`, i.e. it has the opposite sign.
6. The toroidal plasma current given by the input parameter `curtor` and output parameter `ctor` is positive if the current points in the $\nabla \zeta$ direction (counter-clockwise when viewed from above.) This current satisfies

$$\text{curtor} = \int_0^s ds' \int_0^{2\pi} d\theta |\sqrt{g}| \mathbf{j} \cdot \nabla \zeta, \quad (3)$$

where \mathbf{j} is the current density, and the absolute value must be included around the $|\sqrt{g}|$ factor.

7. In the expression for the magnetic field

$$\mathbf{B} = \nabla\psi \times \nabla(\theta + \lambda) + \iota\nabla\zeta \times \nabla\psi, \quad (4)$$

the sign of the ψ , which is the toroidal flux divided by 2π , differs from the sign of vmec's `phi` by `signgs = -1`.