BSCI 474

Fall 2008

Lab #3

Matrix Algebra and Structured Population Dynamics Using MATLAB

Please answer the questions below on a separate sheet of paper or in a separate Word file. Please do not answer the questions solely in your Matlab file. If you use a separate Word file, please name your file the same way your Matlab file (below) is named, but ending in .doc.

1) Open Matlab, and create a new, blank Matlab file named BSCI474_Lab*n*_*XYZ*.m (where *n* is the lab number and *XYZ* are your initials; please use your middle initial, if you have one). The easiest way to do this is to type "edit BSCI474_Lab*n*_*XYZ*.m" in the Matlab command window and save the file once you have typed a few things into it. In the top line, as a "comment" (e.g. after a %) put your name, BSCI 474, the lab number, and the date. As you enter and use the commands that you use below, copy the commands you have used successfully into this file.

2) Consider a Leslie Model for a hypothetical insect with three life stages: egg, larva, and adult. Our insect is such that individuals progress from egg to larva over one time step and from larva to adult over the next time step. Finally, adults lay eggs and die in one more time step. The time step ΔT is 20 days.
   a) Display the text "Question 2" using the Matlab command "`disp('Question 2')`". Copy this command into your Matlab file.
   b) What is the size of the column vector *x* describing the population at any given time? What is the size of the matrix (*P*) used to compute the column vector describing the population at the next time step?
   c) Set the variable "`DeltaT`" to be ΔT. The variable can only hold the numeric value (20), not the units (days), so follow the command with a comment that states its units (days). Copy this command into your Matlab file. *From here on, copy all successfully used commands into your Matlab file.*
   d) The Leslie parameters used to describe the dynamic of a population of this hypothetical insect are: fractional adult birthrate per timestep $f_3 = 73$; percentage survival rate from egg to larva $\tau_{1,2} = 4\%$; percentage survival rate from larva to adult $\tau_{2,3} = 39\%$. Create in Matlab the Leslie matrix *P* that corresponds to this Leslie model.
   e) For an initial population *x*0 of 100 eggs, 0 larvae, 0 adults, what is the expected population *x*1 after 20 days? The expected population *x*2 after 40 days? The expected population *x*3 after 60 days? (Fraction results are allowed since the results of the model are only supposed to be true on average.)
   f) Why are there no larvae or adults after 60 days? Given the birth and survival rates, can you calculate

directly the expected number of eggs in 60 days for the initial conditions, *without matrices*?

g) For an initial population $x0$ of 0 eggs, 0 larvae, 100 adults, what is the expected population $x1$ after 20 days? The expected population $x2$ after 40 days? The expected population $x3$ after 60 days?

h) Why are there no eggs or larvae after 60 days?

i) What is $P*P$? What is $P*P*P$? Explain how this is consistent with both values of $x3$ above, and how it is consistent with the answers to (f) and (h).

3) Consider a Usher Model for a very similar hypothetical insect with three life stages: egg, larva, and adult. This insect is such that individuals progress from egg to larva over one time step and from larva to adult over the next time step, but not all adults die after laying eggs. The time step ΔT is still 20 days.

a) Display the text "Question 3" using the Matlab command "`disp('Question 3')`".

b) What is the size of the column vector x describing the population at any given time? What is the size of the matrix (P) used to compute the column vector describing the population at the next time step?

c) Set the variable "`DeltaT`" to be ΔT. Follow the command with a comment that states its units.

d) The Usher parameters describing the dynamics of a population of this hypothetical insect are: fractional adult birthrate per timestep $f_3 = 73$; percentage survival rate from egg to larva $\tau_{1,2} = 4\%$; percentage survival rate from larva to adult $\tau_{2,3} = 39\%$; percentage survival rate of adults $\tau_{3,3} = 65\%$. Create in Matlab the Usher matrix P that corresponds to this Usher model.

e) For an initial population $x0$ of 100 eggs, 0 larvae, 0 adults, what is the expected population $x1$ after 20 days? The expected population $x2$ after 40 days? The expected population $x3$ after 60 days? This is not as simple as in the Leslie model, but shouldn't look dramatically more complicated.

f) For an initial population $x0$ of 0 eggs, 0 larvae, 100 adults, what is the expected population $x1$ after 20 days? The expected population $x2$ after 40 days? The expected population $x3$ after 60 days? For this initial population (instead of the one in (e)), the results may look dramatically more complicated. This is more typical for a matrix model of population dynamics.

g) What is $P*P$? What is $P*P*P$? Does it get any simpler as it is multiplied more and more?

h) If, for a matrix P, there is a population vector x such that $Px \propto x$, then x is called an *eigenvector* of P, and the proportionality constant is called its *eigenvalue*. Of course for most population vectors x, this is not true. Show that the initial population proportional to $x0$ of 99.9374 eggs, 3.0474 larvae, 1.7958 adults is an eigenvector of the matrix P. What is its eigenvalue? Knowing only its eigenvalue, can you compute the population $x20$ (at 400 days) *without using any matrix algebra*?