**Problem 5.17.** (Magnetic systems.)

**(a)** Faraday's law relates the back-emf in the coil (against which we must do work) to the time rate of change of the magnetic flux. At any moment the magnetic flux is $\Phi_B = NAB$, where $A$ is the cross-sectional area of the coil. According to Faraday's law, therefore, the magnitude of the back-emf is

$$\text{emf} = \mathcal{E} = \frac{d\Phi_B}{dt} = NA\frac{dB}{dt},$$

and so the power that we must supply is

$$\text{power} = \mathcal{E}I = NIA\frac{dB}{dt} = \mathcal{H}LA\frac{dB}{dt} = \mathcal{H}V\frac{dB}{dt},$$

since $\mathcal{H} = NI/L$. To obtain the total energy (work) required for an infinitesimal change in the current, we integrate the power over time to obtain

$$\text{work} = \mathcal{H}V\int\frac{dB}{dt}\,dt = \mathcal{H}V\,dB.$$

**(b)** From the definition of $\mathcal{H}$, we can write

$$B = \mu_0(\mathcal{H} + M/V),$$

and hence

$$dB = \mu_0(d\mathcal{H} + dM/V).$$

The result of part (a) is therefore

$$\text{work} = V\mathcal{H}\mu_0(d\mathcal{H} + dM/V) = \mu_0 V\mathcal{H}\,d\mathcal{H} + \mu_0\mathcal{H}\,dM.$$

In the first term we can write $\mathcal{H}\,d\mathcal{H} = d(\frac{1}{2}\mathcal{H}^2)$, so this term is the change in the quantity $\frac{\mu_0}{2}V\mathcal{H}^2$. If there were no specimen inside the solenoid, this term would give the change in the vacuum field energy; *with* the specimen, $\mathcal{H}$ is the same as without, so this term represents the work we would have to do to increase the field if there were no specimen. If we define the work done on the "system" to exclude this term but include everything else, then

$$W = \text{work done on system} = \mu_0 \mathcal{H}\, dM.$$

(c) The work done on a mechanical system is $-P\, dV$. Apparently, the analogous term for a magnetic system is $+\mu_0 \mathcal{H}\, dM$. The thermodynamic identity for a magnetic system should therefore be

$$dU = T\, dS + \mu_0 \mathcal{H}\, dM.$$

(d) The magnetic analogue of the enthalpy would be

$$H_m = U - \mu_0 \mathcal{H} M,$$

in analogy with the ordinary enthalpy $H = U + PV$. An infinitesimal change in $H_m$ can then be written

$$dH_m = dU - \mu_0 \mathcal{H}\, dM - \mu_0 M\, d\mathcal{H} = T\, dS - \mu_0 M\, d\mathcal{H},$$

where I've used the thermodynamic identity for $U$ in the last step. Interpretation? Apparently the quantity $H_m$ is less than the "system" energy (at least for our situation), and is the more natural "energy" function to use when a process takes place at constant $\mathcal{H}$. To obtain the magnetic analogue of the Gibbs free energy, we can subtract $TS$ just as for a mechanical system:

$$G_m = H_m - TS.$$

Under an infinitesimal change in conditions,

$$dG_m = dH_m - T\, dS - S\, dT = -S\, dT - \mu_0 M\, d\mathcal{H}.$$

Presumably, $G_m$ is the energy that can be extracted as work when the system is held at constant $T$ and constant $\mathcal{H}$ (whereas the Helmholtz free energy, $F = U - TS$, would give the available work in a process at constant $T$ and $M$). [The references given in the text provide further interpretations of the various energy functions for a magnetic system.]