Phys 404 Midterm 1 Solution

TA: Jen-Chien Chang

1.

$S_i = k \ln \Omega_i$

$S_f = k \ln \Omega_f$

$\Delta S = S_f - S_i = +0.1\ J/k = k \ln \frac{\Omega_f}{\Omega_i} \Rightarrow \frac{\Omega_f}{\Omega_i} = \exp[\frac{\Delta S}{k}]$

The probability of reverse process is

$$P = \frac{\Omega_i}{\Omega_f} = \left(\frac{\Omega_f}{\Omega_i}\right)^{-1} = \exp[-\frac{\Delta S}{k}] = \exp\left(-\frac{0.1}{1.38\times10^{-23}}\right) = \underline{\exp(-7.24\times10^{21})}$$

2.

(a) molar specific heat for monatomic ideal gas is

$C_v = \frac{3}{2}R$

(b) net work done by the gas in one cycle = Area of the circle

$\Rightarrow W_{ABCDA} = 1^2\pi = \pi\ (10^9\ dyne \cdot cm) = \underline{\underline{100\pi}}\ (J)$

(c) Internal energy is a function only depends on temperature

$U = nC_vT$, $\Delta U = nC_v\Delta T = \frac{3}{2}nR\Delta T$ (for monatomic ideal gas)

$\Rightarrow \Delta U_{AC} = \frac{3}{2}\cdot 1\cdot R\cdot\left(\frac{P_cV_c}{1\cdot R} - \frac{P_aV_a}{1\cdot R}\right) = \frac{3}{2}(6-2) = 6\ (10^9\ dyne\cdot cm) = \underline{\underline{600\ (J)}}$

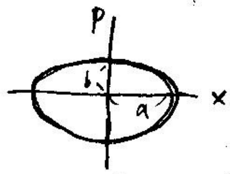
(d) first law of thermodynamics : $\Delta U = \Delta Q - W$ (note that ΔQ and W are path-dependent)

W_{ABC} = area below curve ABC = $4 + \frac{\pi}{2}\ (10^9\ dyne\cdot cm)$ … work done by the gas

ΔU_{AC} is path dependent , $\Delta U = 6\ (10^9\ dyne\cdot cm)$

$\Rightarrow \Delta Q = \Delta U + W = \underline{(10 + \frac{\pi}{2})\ (10^9\ dyne\cdot cm)} \approx \underline{1157\ (J)}$

3. (a) $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{p^2}{(\sqrt{2m})^2} + \frac{x^2}{\left(\sqrt{\frac{2}{m\omega^2}}\right)^2}$



for 1-d quantum harmonic oscillator, the curve of equal energy on the phase space (2-d) is an ellipse, with $\begin{cases} a = \sqrt{2mE_n} \\ b = \sqrt{\frac{2}{m\omega^2}E_n} \end{cases}$

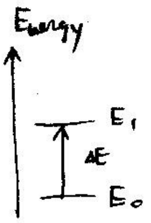
$\Rightarrow$ "Volume" of phase space $\int dp\,dx$ = area of the ellipse $= \pi ab$

$\Rightarrow$ $V_n = \pi\sqrt{2mE_n}\sqrt{\frac{2E_n}{m\omega^2}} = \frac{2\pi E_n}{\omega} = \frac{2\pi(n+\frac{1}{2})\overset{\frac{h}{2\pi}}{\hbar}\omega}{\omega} = (n+\frac{1}{2})h$

V_n is the volume containing n microstates

$\therefore$ $V_1 = V_{n+1} - V_n = \underline{\underline{h}}$

(b)



$\text{fraction} = \frac{6e^{-\left(\frac{E_0+\Delta E}{kT}\right)}}{2e^{-E_0/kT} + 6e^{-\frac{(E_0+\Delta E)}{kT}}} \xlongequal{\text{divided by } e^{-\frac{E_0}{kT}}} \frac{3e^{-\frac{\Delta E}{kT}}}{1+3e^{-\frac{\Delta E}{kT}}} = \frac{1}{\frac{1}{3}e^{\frac{\Delta E}{kT}}+1}$

E_0 → energy of ground states

ΔE → energy difference between gnd state and first excited state

$\Delta E = 1.2\,(eV) = 1.2 \times 1.6 \times 10^{-19}$ (J).

$\Rightarrow \exp\left(\frac{\Delta E}{kT}\right) = \frac{1.2 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 6000} \approx 10$

$\Rightarrow$ fraction $\approx \frac{1}{\frac{10}{3}+1} \approx 0.23$

(Conceptually not correct to set $E_0 = 0$)

4. See hw 3, problem 1