1 (from Prof Hu)

Phys 404 - HW 1

Problem 1

a.) (i) 3 possible configurations (microstates)

(ii) 9 microstates

(iii) 27 "

b.) (i) 4 "

(ii) 16 "

(iii) 64 "

c) $$\# \text{ of microstates} = (\# \text{ of partitions})^{\# \text{ of particles}} = n^N = \left(\frac{V}{v}\right)^N$$

Problem 2

a.) Regardless of whether N is even or odd, we can expand the symbolic representation $(\uparrow + \downarrow)^N$ as on p. 14

$$(\uparrow + \downarrow)^N = \sum_{t=0}^{N} \frac{N!}{(N-t)!\,t!} \uparrow^{N-t} \downarrow^{t}$$

Also, we can again replace t with $N_\downarrow$ (where $N = N_\uparrow + N_\downarrow$) and get

$$(\uparrow + \downarrow)^N = \sum_{N_\downarrow = 0}^{N} \frac{N!}{N_\uparrow!\, N_\downarrow!} \uparrow^{N_\uparrow} \downarrow^{N_\downarrow}$$

The book uses the definitions $N_\uparrow = \frac{1}{2}N + s$ and $N_\downarrow = \frac{1}{2}N - s$ to rewrite this in the form of equation (14)

$$\boxed{(\uparrow + \downarrow)^N = \sum_s \frac{N!}{(\frac{1}{2}N + s)!\,(\frac{1}{2}N - s)!} \uparrow^{\frac{1}{2}N + s} \downarrow^{\frac{1}{2}N - s}}$$

This is still appropriate, as long as we realize that s will have different values for odd N

B) By definition, $s = \frac{N_\uparrow - N_\downarrow}{2}$

For odd N, $N_\uparrow - N_\downarrow$ will always be an odd integer. Thus s will always be half-integer values

e.g. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2},$ etc.

Possible values of s are

$$s = -\frac{N}{2}, -\frac{N}{2}+1, -\frac{N}{2}+2 \cdots -\frac{1}{2}, \frac{1}{2}, \frac{3}{2} \cdots \frac{N}{2}-1, \frac{N}{2}$$

C) For N=5, possible s values are

$-\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$

$$g = \frac{5!}{\left(\frac{5}{2}+s\right)!\left(\frac{5}{2}-s\right)!}$$

s	g
$-\frac{5}{2}$	1
$-\frac{3}{2}$	5
$-\frac{1}{2}$	10
$\frac{1}{2}$	10
$\frac{3}{2}$	5
$\frac{5}{2}$	1