Physics 404 (Prof. Hu) Final Examination Dec. 18, 2007

(Total 7 problems, 200 points. Write YOUR NAME on your exam book NOW!)

1. (24 pts) Consider a non-interacting gas of spin 1/2 fermions each of mass m initially confined in a volume V_i at temperature $T = 0$. Let this gas expand irreversibly into a vacuum to a final volume of V_f. Calculate the temperature of the gas after expansion if V_f is sufficiently large for the classical limit to apply. (12 pts) By what factor of expansion does the gas need to undergo so that its temperature will settle into a constant final value? (12 pts)

2. (30 pts) A container is divided into two equal parts by a partition containing a small hole of diameter D. Helium gas in the two parts is held at temperature $T_1 = 100K$ and $T_2 = 400K$ respectively through heating of the walls.

a) How does the diameter D determine the physical processes which bring the system into a steady state? State the working principles behind each case you consider. (10 pts)

b) What is the ratio of the mean free paths λ_1/λ_2 between the two parts when $D \ll \lambda_1, \lambda_2$ and the system has reached a steady state? (10 pts)

c) What is the ratio λ_1/λ_2 when $D \gg \lambda_1, \lambda_2$? (10 pts)

3. (25 pts) Consider an ideal Bose gas at temperature T of N particles each of mass m confined to a line of length L. Show that no condensate can be formed. (Hint: Show that the integral for the number of particles in excited states $N_e(T)$ does not converge.) Explain what this means physically.

4. (36 pts) a. Calculate the Fermi energy ϵ_F of a gas of N extreme relativistic electrons (where the energy $\epsilon = pc$ momentum x speed of light) in a volume V (number density $n \equiv N/V$). (8 points)

b. Calculate the total energy U_0 of this gas at its ground state in terms of ϵ_F. (8 points)

c. Use these results to derive the number of electrons N in a white dwarf star of mass M and radius R. Make the assumption that the whole star is ionized hydrogen with mass m_H (equal in number to the electrons), but neglect the kinetic energy of the protons compared to that of the electrons. You may invoke the Virial Theorem. Recall the gravitational potential (self) energy is $\frac{3GM^2}{5R}$ where G is Newton's constant. (20 points)

5. (20 pts) Derive (don't just copy it from your formula sheet) an expression for the critical radius R_c for the nucleation of a droplet from vapor (of the same chemical composition) in terms of the surface tension (free energy per unit area) σ, the concentration of molecules in the liquid n_l, and the chemical potential difference $\Delta\mu = \mu_g - \mu_l$ between gas and liquid.

6. (25 pts) What is the order of the BEC phase transition (5 points)? Calculate the latent heat released when a BEC turns into a Bose gas at the critical temperature T_c. Express this in terms of kT and the Bose-Einstein functions $g_n(z)$ of order n and argument z (the fugacity of the gas). Hint: Use the Clausius-Clapeyron equation. For a Bose gas at low temperature

its specific volume v (volume per atom) is given by

$$\frac{N}{V} = \frac{1}{v} = \frac{g_{3/2}(z)}{\lambda_{tdB}^3} + \frac{1}{V}(\frac{z}{1-z}) \tag{1}$$

and its pressure at T_c is given by

$$P = \frac{kT}{\lambda_{tdB}^3} g_{5/2}(1) \tag{2}$$

where λ_{tdB} is the thermal de Broglie wavelength of the gas molecule. (20 points)

7. (40 pts) a. In a simple experiment a student pours 700 cm^3 of liquid nitrogen in a dewar sitting on a pan balance and measures that it has a weight of 563 grams. Use the fact that N_2 has molecular weight of 28 (g/mole) to calculate the diameter d (10 points) of the nitrogen molecule and the mean free path (10 points) λ of the nitrogen gas molecules at STP.

b. In a lecture demonstration of diffusion velocity, cotton swabs dipped into HCl and NH_4OH respectively are inserted into the ends of the tube. HCl gas (molecular weight 36.5) and NH_3 or ammonia gas (molecular weight 17) begin to diffuse inward. In less than 15 minutes they meet, forming a ring of ammonium chloride (NH4Cl). Calculate the ratio of distances ℓ_{HCl}/ℓ_{NH3} travelled by each vapor before they meet. (10 points)

c. The diffusivity of a classical gas is $D = \frac{1}{3}v_{rms}^2\tau_c$ where v_{rms} is the root-mean-square velocity of the molecules and τ_c is their mean collision time. Calculate the diffusivity of a Fermi gas. Use physical arguments instead of detailed mathematical derivations. (10 points)

Note: you need to explain every step in your derivations. You won't get any credit if you just copy the results from your formula sheet. The following information may be useful:
Avagadro's number $N_A = 6.022 \times 10^{23}$ per mole.
Boltzmann's constant $k = 1.38 \times 10^{-23} J/K$
Planck's constants $h = 6.63 \times 10^{-34} Joules - sec.$
Define $I(n,a) = \int_0^\infty e^{-ax^2} x^n dx$, then $I(0,a) = \frac{1}{2}\sqrt{\pi}a^{-\frac{1}{2}}$; $I(1,a) = \frac{1}{2}a^{-1}$; $I(2,a) = \frac{1}{4}\sqrt{\pi}a^{-3/2}$

** Happy Holidays! **