Physics 404 (Prof. Hu) Second Midterm Exam Nov. 27, 2007

1. a) N photons are in equilibrium at temperature T in a cavity of volume V. Derive an expression for the number N_e of thermally excited photons. You may find this information useful: $\int_0^\infty \frac{u^2 du}{e^u - 1} = 2.404$ (18 points)

b) Find the critical temperature below which $N_e < N$, assuming a photon density of $10^{20}/cm^3$. The excess $N - N_e$ will be in the photon mode of lowest frequency which together would constitute a condensate. Can you find such a condensate in nature? Why? (7 points)

2. a) What is the meaning of the Debye temperature T_D for a solid? Derive (don't just copy from your formula sheet – explain every step) an expression for T_D for a solid of atom number density n and velocity of sound v_s. As a first step, derive the density of state $D(\nu)$ function for phonons in a solid. (22 points)

b) Explain the T^3 dependence at low temperature of the heat capacity at constant volume C_V of solid due to lattice vibrations from a physically intuitive way invoking the meaning of T_D. (8 points)

3. (30 points) A total of N fermions of spin $\hbar/2$ and mass m are restricted to motion in two dimensions on a plane of area A with no mutual interaction. For temperature below the Fermi temperature, calculate
a) The Fermi energy ϵ_F (6 points)
b) The energy of the ground state E_0 (6 points)
c) The average energy per particle U/N (6 points)
d) The heat capacity at fixed area (6 points)
e) The entropy S (6 points)

4. A surface with N_0 adsorption centers has N gas molecules adsorbed (adhered) on it ($N_0 > N$) at temperature T. Find the chemical potential μ in terms of Z_1, the partition function of a single adsorbed molecule.(15 points)

Avagadro's number $N_A = 6.022 \times 10^{23}$ per mole.
Boltzmann's constant $k = 1.38 \times 10^{-23} J/K$
Planck's constants $h = 6.63 \times 10^{-34} Joules - sec.$