

Problem 5.17. The enthalpy and Gibbs free energy, as defined in this section, give special treatment to mechanical (compression-expansion) work, $-P dV$. Analogous quantities can be defined for other kinds of work, for instance, magnetic work.* Consider the situation shown in Figure 5.7, where a long solenoid (N turns, total length L) surrounds a magnetic specimen (perhaps a paramagnetic solid). If the magnetic field inside the specimen is \vec{B} and its total magnetic moment is \vec{M} , then we define an auxiliary field $\vec{\mathcal{H}}$ (often called simply the magnetic field) by the relation

$$\vec{\mathcal{H}} \equiv \frac{1}{\mu_0} \vec{B} - \frac{\vec{M}}{V},$$

where μ_0 is the “permeability of free space,” $4\pi \times 10^{-7} \text{ N/A}^2$. Assuming cylindrical symmetry, all vectors must point either left or right, so we can drop the $\vec{}$ symbols and agree that rightward is positive, leftward negative. From Ampere’s law, one can also show that when the current in the wire is I , the \mathcal{H} field inside the solenoid is NI/L , whether or not the specimen is present.

- Imagine making an infinitesimal change in the current in the wire, resulting in infinitesimal changes in B , M , and \mathcal{H} . Use Faraday’s law to show that the work required (from the power supply) to accomplish this change is $W_{\text{total}} = V\mathcal{H} dB$. (Neglect the resistance of the wire.)
- Rewrite the result of part (a) in terms of \mathcal{H} and M , then subtract off the work that would be required even if the specimen were not present. If we define W^\ddagger , the work done on the *system*,[†] to be what’s left, show that $W = \mu_0 \mathcal{H} dM$.
- What is the thermodynamic identity for this system? (Include magnetic work but not mechanical work or particle flow.)
- How would you define analogues of the enthalpy and Gibbs free energy for a magnetic system? (The Helmholtz free energy is defined in the same way as for a mechanical system.) Derive the thermodynamic identities for each of these quantities, and discuss their interpretations.

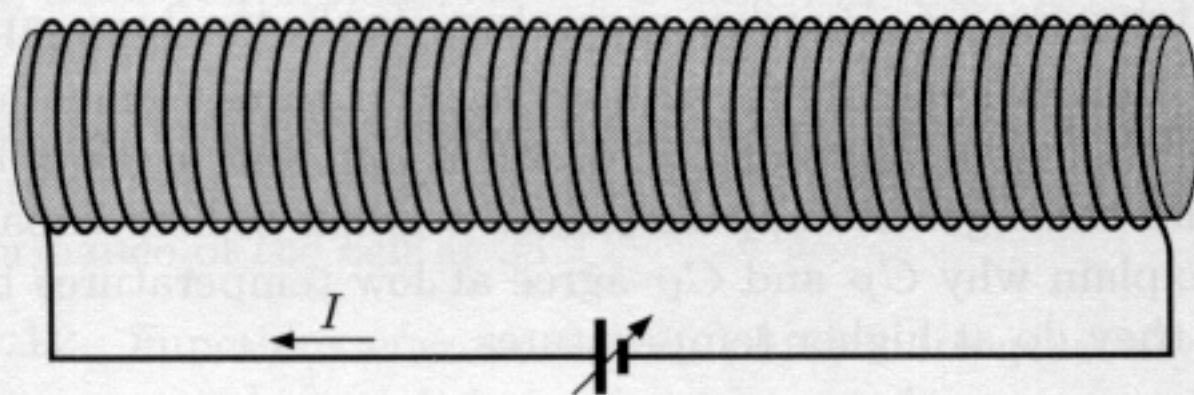


Figure 5.7. A long solenoid, surrounding a magnetic specimen, connected to a power supply that can change the current, performing magnetic work.

*This problem requires some familiarity with the theory of magnetism in matter. See, for instance, David J. Griffiths, *Introduction to Electrodynamics*, third edition (Prentice-Hall, Englewood Cliffs, NJ, 1999), Chapter 6.

[†]This is not the only possible definition of the “system.” Different definitions are suitable for different physical situations, unfortunately leading to much confusion in terminology. For a more complete discussion of the thermodynamics of magnetism see Mandl (1988), Carrington (1994), and/or Pippard (1957).