

Limits on the frequency dependence of the speed of light

Suppose the photon frequency—i.e. the frequency of electromagnetic waves—satisfies the modified dispersion relation:

$$\omega^2 = c^2 k^2 (1 + ak), \quad (1)$$

where c is the usual speed of light in vacuum, and a is a constant with the dimensions of length. If a distant gamma ray burst suddenly emits radiation over a range of frequencies, the different frequency components would arrive at the earth at different times if a is not zero. If it is observed that radiation peaked at two wavenumbers k_1 and k_2 travelling at group velocities v_1 and v_2 over a distance D arrive within a time difference Δt , that gives us an upper limit on the length a .

1. Show that, with the symbols defined above, the difference of arrival times $t_2 - t_1$ would be given by

$$t_2 - t_1 = D \left(\frac{1}{v_2} - \frac{1}{v_1} \right) \simeq \frac{D}{c^2} (v_1 - v_2), \quad (2)$$

where the approximate equality holds when v_1 and v_2 are both very close to c . Show that both the exact and approximate equalities hold.

2. Show that the group velocity $v(k)$ for the dispersion relation (1) is given by

$$v(k) = c \frac{1 + \frac{3a}{2}k}{\sqrt{1 + ak}} \simeq c(1 + ak), \quad (3)$$

where the approximate equality holds when $ak \ll 1$, i.e. when the wavelength is much longer than a . Show that both the exact and approximate equalities hold.

3. If the difference in arrival times of radiation at wavenumbers k_1 and k_2 is observed to be less than Δt , what upper limit can be inferred for a ?
4. Suppose $D = 260$ Mpc, $\Delta t = 25$ ms, and the frequencies of the two observed components of the radiation are $\nu_1 = 7.2 \times 10^{18}$ Hz, and $\nu_2 = 1.1 \times 10^{23}$ Hz. (Here 1Mpc= 10^6 pc, one “megaparsec”, is a commonly used astronomical unit of distance, where one parsec is 1pc = 3.0856×10^{18} cm, or about 3.26 light years. So 260 Mpc is almost a billion light years, a tenth of the way back to the big bang!) What upper limit do you find for a ? (The article by Schaefer quoted something like 10^{-30} cm.)
5. The Planck length L_P is the unique quantity with dimensions of length that can be formed from Newton’s constant G , the speed of light c , and Planck’s constant \hbar . Using dimensional analysis, find the formula for L_P , and find its numerical value. You should get something around 10^{-33} cm. This is interestingly close to the upper limit on a found above. It means that if quantum gravity effects produce a dispersion relation for photons of the form (1), we need only three orders of magnitude improvement in the observational limits to begin to see the effects of quantum gravity.

Source: Physics News Update; The American Institute of Physics Bulletin of Physics News

Number 432 (Story #2), June 7, 1999 by Phillip F. Schewe and Ben Stein

THE SPEED OF LIGHT IS INDEPENDENT OF FREQUENCY within a factor of 6×10^{-21} . Bradley Schaefer of Yale bases this estimate on the observed arrival of gamma rays from distant explosive events in the cosmos, such as gamma-ray bursters. If the speed of light (c) were slightly different for the different frequency ranges, then some light waves would show up before the others, but this is not the case. The best previous effort to locate a frequency dependency for c , deduced from light coming from the Crab pulsar, was at the 5×10^{-17} level. Why would c vary with frequency? Einstein's theory of relativity, and its insistence on a universal light speed, might be at fault. Or photons might have mass. Schaefer's analysis addresses this issue, and puts an upper limit of 10^{-44} g on any putative photon mass, not quite as sharp a limit as those based on the observed strength of the galactic magnetic field (a nonzero photon mass would allow the fields to decay away). The new sharper limits on any possible frequency-dependency for c is a vindication of relativity. By the way, the prefix for anything as small as 10^{-21} is "zepto" (Shaefer, Physical Review Letters, 21 June.)

And the abstract of the actual paper:

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Severe Limits on Variations of the Speed of Light with Frequency

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Explosive astrophysical events at high redshift can be used to place severe limits on the fractional variation in the speed of light with frequency ($\Delta c/c$), the photon mass (m_γ), and the energy scale of quantum gravity (EQG). I find $(\Delta c)/c < 6.3 \times 10^{-21}$ based on the simultaneous arrival of a flare in GRB 930229 with a rise time of 22030 s for photons of 30 and 200 keV. The limit on m is 4.2×10^{-44} gm for GRB 980703 from radio to gamma ray observations. The limit on EQG is 8.3×10^{16} GeV for GRB 930131 from 30 keV to 80 MeV photons.