EXAM#2 —Phy273—Fall 2000 Prof. Ted Jacobson
Monday, Nov. 20, 2000 Room 4117, (301)405-6020
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If you miss an earlier part of a problem do not give up on later parts of the problem, even if they require the result of the earlier part. You can get partial credit by just solving the later part in a more general way.

1. The pressure p in a compressional wave travelling through oobleck (remember Dr. Seuss?) takes the form

$$p(x, y, z, t) = \frac{p_0}{(\alpha x + \beta + \gamma t)^2 + \delta^2}, \tag{1}$$

where $\alpha, \beta, \gamma, \delta$ are constants. What is the wave speed and direction of travel? [15 *points*]

2. A guitar string of length 50 cm and linear mass density 1 gm/m is tuned to a fundamental frequency of 300 Hz.

(a) Sketch the displacement amplitude in the fundamental mode, and indicate the location of the displacement nodes and antinodes. [5 *points*]

(b) Suppose the string vibration in the fundamental mode is "linearly polarized", with a maximum displacement amplitude of 1 mm. Write an expression $y(x, t)$ for the string displacement as a function of space and time. [15 *points*]

(c) What point on the string attains the maximum transverse speed, and what is that speed? [5 *points*]

(d) What is the total energy in the vibration of question 2b? (*Hint*: When the amplitude vanishes the energy is all kinetic.) [5 *points*]

(e) Suppose the string is anchored at one end to a post. What is the force $\vec{F}(t)$ on the post in the vibrational mode of question 2b? (Let the string run from coordinates $(0, 0, 0)$ to $(L, 0, 0)$, with the post at the origin, and let the direction of polarization be $\hat{y}$. Make sure you include both the constant and time dependent parts of the force, and include both magnitude and direction information. Make the usual small displacement approximation.) [5 *points*]

3. Consider a cylindrical tube of length 20 cm, closed at one end and open at the other.

(a) Sketch the pressure amplitude as a function of position in the first two resonant modes of the tube for sound in air, and indicate the location of the pressure nodes and antinodes. (Make sure you indicate clearly which is the closed end and which is the open end.) [10 *points*]

(b) What are the frequencies of these modes? [15 *points*]

4. The wave equation for waves on a string with some stiffness (resistance to bending) has the form

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} - \alpha \frac{\partial^4 y}{\partial x^4}. \tag{2}$$

(a) What are the dimensions of α? [5 *points*]

(b) Find the dispersion relation $\omega = \omega(k)$ for waves satisfying this equation. (Use the method of substituting in a trial function of the form $y(x, t) = y_0 \exp(ikx - i\omega t)$.) [5 *points*]

(c) What are the phase and group velocities for these waves? [5 *points*]

(d) What condition should be satisfied by the wavenumber k in order for the stiffness to make only a very small correction compared to the case with no stiffness? [5 *points*]

(e) What is the *approximate* dispersion relation, including only the lowest order correction due to the stiffness? [5 *points*]