Practical Implementation of a Traffic Congestion System

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Abstract

This paper studies traffic congestion abatement as a practical implementation problem. In theory, a social planner can ensure efficient behavior by setting the price faced by decision makers to be equal to the social marginal cost. In practice, however, policy makers lack the information to set such a price. This paper develops a decentralized and anonymous dynamic system that allows agents to achieve the optimal level of traffic without knowledge of demand or social marginal cost. An experiment was used to test the effectiveness of the system. In the lab, the system achieved an efficiency of 95%.

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Congestion is an important source of economic waste. Empirical studies have found that the loss of welfare due to traffic congestion is between $32 and $121 billion dollars every year in the United States;\(^1\) accounting for 5.5 billion hours of extra travel time and 2.9 billion gallons of extra fuel. Congestion costs are the product of socially inefficient behavior by drivers who do not take into

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\(^1\)According to Schrank et al. [2012] the congestion “invoice” for the cost of extra time and fuel in 498 urban areas in 2011 was (in 2011 dollars): $121 billion. On the other hand, Litman [2014] considers that $32 is a more appropriate value, as the former report consider a value of time “unreasonable” high. The value of time consider by the former is $16.79 per hour and $8.37 by the latter. Theses studies have also a different position on the efficient level of congestion.
account the congestion they produce when deciding their travel route or whether to drive at all. In theory, a social planner could ensure efficient behavior and no waste by introducing a congestion price. If drivers faced a congestion price equal to the value of the marginal congestion they produce at the socially optimum level of congestion, the social optimum could be achieved. In practice, however, no policy maker has this information.

This lack of information is a problem that no system has been able to solve in practice. For example, both the Congestion Charge in London and Singapore’s Area Licensing Scheme, which are deemed the most successful congestion systems in the world, use demand estimations and an objective level of congestion to set the congestion price to be charged to drivers.\(^2\)

This study is concerned with the practical implementation of a traffic congestion system. It is assumed that policy makers have no knowledge about the value of the marginal social cost of driving or the benefit from driving. In addition, it is assumed that policy makers face political constraints such as revenue neutrality, anonymity and lack of binding contracts. In this system, drivers will report the value of their time, but will make driving decisions by themselves. The system will use the reports made by agents to calculate an appropriate combination of taxes and subsidies at any given level of traffic. In theory, under certain behavioral assumptions, the system is guaranteed to achieve the social optimum. However, the validity of these assumptions and the effectiveness of the system are empirical questions; here studied using an experiment.

The design of the congestion system is an implementation problem in which a social planner faces hidden information and hidden actions. It is assumed that the social planner does not know the value of time of each individual traveler or the value derived from commuting from one point to another. In addition, it is assumed that the social planner has only a limited capacity to observe individual actions and enforce contracts. The second assumption rules out well-known implementation mechanisms such as the Vickrey-Clarke-Groves mechanism (Vickrey [1961], Clarke [1971], Groves [1973]) and dominant implementation in general.

In the VCG mechanism, the lack of information is circumvented by having agents send reports to the mechanism. After reports have been received, the mechanism implements an efficient allocation and

\(^2\)Z.F. Li [1999] describes the evolution the the Singapore’s Area Licensing Scheme, which originally had a target reduction of 25% - 30%. According to the transport for London report [2003], the London’s congestion charging was originally intended to reduce traffic by 10% - 15%.
collects payments from all agents. Payments are set such that it is a dominant strategy for all agents to report truthfully. However, if the mechanism has no power to implement the efficient allocation or lacks the ability to sign and enforce binding contracts, then dominant implementation is lost. Suppose that, in the efficient allocation, there are two identical individuals of whom only one is supposed to drive. In the absence of binding contracts, what could prevent the driver who is not supposed to drive from driving? After all, there is an identical driver who found optimal to drive even after covering the payment imposed by the mechanism. Even if the social planner had implementation power, there are two characteristics that would make the VCG mechanism difficult to implement in real life: i) agents taking identical actions could face different payments, ii) agents would need to relinquish their decisions to the mechanism, requiring high levels of trust, and iii) the mechanism requires massive amounts of computational power.

When dominant implementation is impossible, Nash implementation is usually the second choice. However, the latter solution concept requires rationality assumptions of an order of magnitude higher than the former. These assumptions are, at best, difficult to defend when there is a huge number of agents, such as in our object of study. In particular, agents might not have information on each others’ payoff functions, agents might fail to best respond to their current information and beliefs, and beliefs among agents might not be consistent. On the other hand, even when agents have no information on the number, identity, payoffs or beliefs of other agents, it is sensible to assume that they will try to improve their payoffs over time by means of trial and error. This latter assumption is the basis of evolutionary game theory and hence we focus on evolutionary implementation as the method to design our system.

We assume that agents adjust their behavior over time and that they respond only to the current incentives they face. Individual responses do not need to be “best responses” and we focus on aggregate responses only. It is shown that if aggregate responses are correlated with current aggregate payoffs and reports are balanced (on average, reports represent the true average value of time among current drivers), then the system converges to the social optimum. On the other hand, it is clear that “drive no matter the cost” is a behavior that would prevent any system from achieving the

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3For example, Aumann and Brandenburger [1995] showed that when the number of players is at least three it is necessary to assume that there is a common prior, mutual knowledge of the payoff functions, mutual knowledge of rationality, and common knowledge of the conjectures for a Nash equilibrium to be played.
optimal level of congestion. Both behavioral assumptions might or might not hold in practice. We test these assumptions in the lab and measure the general effectiveness of the system.

Our experimental design consists of a driving game in which 14 subjects independently decide whether to “drive” or “not drive” on a fixed road for 30 rounds of play. At the beginning of the game, every subject was randomly and privately assigned two numbers: i) value of commuting and ii) value of time. Neither the distribution nor the support of values was revealed to the subjects. The set of values was chosen to minimize the set of drivers who belong to both the optimal allocation and the equilibrium with no congestion pricing. The equilibrium with no congestion pricing consists of 10 drivers and the optimal allocation consists of 6, however, only two drivers belong to both allocations. In other words, 12 out of 14 drivers would have to change their behavior with the introduction of the congestion control system. We believe that such a radical change in the allocation is a strong test for the effectiveness of the system.

In order to assess the general effectiveness of the system and the relative costs of losing information from the policy maker’s perspective we considered 6 treatments: i) No tax - the benchmark to measure the level of congestion; ii) Fixed tax - the optimal fixed tax assuming the policy maker knows the demand and marginal social cost; iii) Dynamic tax - it is assumed that the policy maker knows the marginal social cost, but not the demand; iv) Message tax - this treatment represents the system we are proposing, which assumes that the policy maker does not know the demand or the marginal social cost. The last two treatments are identical to the dynamic and message treatments with the additional restriction that the system needs to be revenue neutral.

Our experimental results are promising. We measure efficiency with respect to the observed efficiency achieved by the fixed-tax treatment. This treatment represents the maximum possible efficiency a policy maker could achieve in a real situation. The observed efficiencies are as follows: 65.90% for no-tax; 100.00% for dynamic; 94.99% for message; 101.05% for balanced dynamic; and 94.35% for balanced message. Our experimental results are relevant in two ways. First, they show that high efficiency levels can be implemented in real life without knowledge of demand or marginal social cost. Second, creating a revenue neutral system is possible in practice. In theory, when all

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4The Mann-Whitney test was used to reject the null hypothesis that the treatments with tax achieve the same efficiency as the treatment with no tax. In all cases the null was rejected at a confidence level of 99%. The same test was to used to test the relative efficiency of all tax treatments. In all cases the null (the treatments achieve the same efficiency) was not rejected.
agents best respond and anticipate correctly the consequences of their actions, achieving high efficiency and revenue neutrality are impossible as the latter distorts incentives to report information truthfully.

This paper is closely related to the literature on congestion abatement systems, and the literature on theoretical and experimental convergence to equilibrium. Externalities and externality abatement have been studied consistently at least since Pigou [1920] who proposed to charge agents the value of the marginal externality they produce at the efficient social allocation. As mentioned before, this approach requires the policy maker to know the demand, the marginal social cost, and the efficient social allocation. Several approaches have been proposed in the literature to deal with the case in which the demand is unknown, but the social cost is known. These approaches exploit convergence properties of dynamic systems to discover the efficient social allocation.

Sandholm [2002, 2005, 2010] provides a systematic study of this approach. Sandholm considers a model in which the value of externalities is homogeneous across agents and known to the policy maker. He shows that charging the current value of the marginal social cost leads the system to the social optimum as long as the average aggregate behavior is correlated with current payoffs.

Both Li [2002] and Yang et al. [2004] provide evidence that the above procedure converges not only when the adjustment is continuous and changes in behavior are slow over time but also when changes are made in discrete time and agents best respond at all moments to current conditions.

Yang and Wang [2011] study systems of tradable permits. They show that the system can achieve full efficiency when the market for permits is perfectly competitive. Continuing their work, Wang et al. [2014] showed that the system of tradable permits can be guaranteed to achieve the social optimum allocation by adjusting the quantity of permits according to the observed price in the permits market. Nie [2012] have shown that these tradable permit systems are very sensitive to transaction costs in the permits market. Guo and Yang [2010] show that, when demand is fixed, it is possible to achieve revenue neutrality using an appropriate combination of taxes and subsidies.

Our study is closely related to the model of Sandholm; however we do allow for heterogeneous valuations of the externality i.e. different people have possibly different values of time. Furthermore, we do not assume that the policy maker knows the marginal social cost and hence its discovery is also part of the dynamic process. These deviations come at a price. In Sandholm’s model, the system is
guaranteed to converge to the social optimum under a wide class of behavioral assumptions. In our model, a smaller class is considered. However, our experiment shows that actual behavior belongs to the smaller class. Within the evolutionary game theory tradition, it is often argued that revision protocols\(^5\) are the source of aggregate behavior; however, it has been shown (Chen and Gazzale [2004], Healy [2006], Cominetti et al. [2010], Melo [2011]) that learning processes also produce aggregate behavior consistent with convergence to equilibrium. We are completely agnostic about the several possible reasons for why the aggregate behavior should converge to equilibrium. In addition, we show that revenue neutrality is possible even when demand is elastic without the help of an additional permits market.

Several studies have taken congestion games to the experimental lab to study entry (our first treatment), route choice behavior (we do not study route choice) and the effectiveness of fixed taxes (our second treatment). Schneider and Weimann [2004], Selten et al. [2007], and Hartman [2012] study route choice behavior with and without congestion prices. Rapoport et al. [2004], Anderson et al. [2007], and Rapoport et al. [2014] study entry games with and without congestion prices. These studies establish a robust characteristic of congestion games: on average (over time) the equilibrium quantity of drivers is observed, but there are persistent fluctuations around equilibrium. In these studies, the persistent fluctuations are produced by the homogeneity of agents values; there is a single equilibrium quantity of drivers, but there are multiple possible subsets of drivers consistent with this equilibrium quantity. Our design rules out this multiplicity of equilibria for two reasons. On the one hand, time values and commute values are not likely to be homogeneous in the real word, specially when large numbers of drivers are considered. On the other hand, both our first and second treatments are used as controls to test the effectiveness of our system. Fluctuations around equilibrium would decrease the efficiency of the benchmarks and consequently increase the apparent gains of our system. In our experiment, no two agents are equal in terms of values and the equilibrium is unique for the first and second treatments.

\(^{5}\)A revision protocol is a rule followed by agents to select their actions; when aggregated, revision protocols define the aggregate behavior of the system. Evolutionary game theory traditionally classifies aggregate behavior into four categories: imitation, excess payoff, pairwise comparison, and best responses. See Sandholm [2010] for an extensive review of revision protocols.
1 Traffic congestion model

Real life traffic congestion occurs when thousands of drivers use a road network. During congested times, the marginal effect of each individual on the total congestion is very small, but the total effect can be large. In general, many drivers use rules of thumb when deciding where or when to drive. For example, an agent may choose to commute from home to his office using the same route and the same departure time every day. Drivers do not know each other, and do not coordinate routes or departure times. We believe these characteristics are better captured by the concepts and tools of evolutionary game theory. In this section we define the basic concepts of the model.

A continuum of agents want to commute using a single road during a single peak time of the day. The total time spent by each agent commuting is a function of the number of agents on the road and is characterized by a strictly increasing, twice differentiable function $t : \mathbb{R} \rightarrow \mathbb{R}_+$. There is a finite set of types $R$, with typical element $r \in R$ and mass denoted by $m_r$. We denote the set of agents of type $r$ as the population $r$ and the whole set of agents as the population. Every type is characterized by an inverse demand function $D_r : [0, m_r] \rightarrow [0, U_r]$ and a fixed value of time $v_r$. It is assumed that $D_r$ is differentiable and strictly decreasing. $D_r(x_r)$ is the value derived from commuting by the $x_r$-th member of population $r$. All types have an outside option with value $0$.

Outcomes are identified by strategy distributions $x \in X = \{x \in \mathbb{R}_+^R : x_r \leq m_r\}$, where $x_r$ represents the mass of agents of type $r$ who drive. The payoff function for population $r$ is denoted by $F_r : X \rightarrow \mathbb{R}$ and is given by $F_r(x) = D(x_r) - v_r t(x)$; where $x = \sum_{r \in R} x_r$ is the total mass of drivers. We use $x$ to denote both the total number of drivers on the road and the strategy distribution, we believe there is no risk of confusion. The collection of payoff functions is denoted by $F : X \rightarrow \mathbb{R}^{|R|}$. A strategy distribution $x \in X$ is a Nash Equilibrium if all players maximize their payoffs given the strategy distribution: $F_r(x) \geq 0$ whenever $x_r > 0$ and $F_r(x_r, y_r) < 0$ for every $y_r > x_r$, $y_r \in [0, m_r]$. We assume that $t$ and $D_r$ for all $r \in R$ are such that an interior Nash equilibrium always exists.

When a Nash Equilibrium is specified, it is in the best interest of all players to follow it. Conversely, when a non-equilibrium strategy is specified, at least one player can gain by changing his strategy. However, it is not clear if and when a sequence of non-equilibrium strategies and their respective de-

\textsuperscript{6}A generalization of the model for a general road network and departure time choice is included in the appendix.
viations actually lead to a Nash Equilibrium. Thus characterizing non-equilibrium behavior is essential to study the convergence properties of the model. We introduce mean dynamics and Lyapunov functions to do so.

A mean dynamic $V : X \to \mathbb{R}^{|R|}$ is a function that defines an equation of motion $\dot{x} = V(x)$ on the space of strategy distributions. We say that $V$ has a Lyapunov function if there is $Y$ open neighborhood of $X$, and $L : Y \to \mathbb{R}$ such that $\nabla L(x)^t V(x) \leq 0$ for all $x \in X$.

$V$ is called admissible with respect to $F$ if:

- $V$ is Lipschitz continuous.
- $V_r(x) \geq 0$ whenever $x_r = 0$.
- $V_r(x) \leq 0$ whenever $x_r = m_r$.
- $V(x) = 0$ implies $x$ is a Nash Equilibrium of $F$.
- $V$ has a Lyapunov function.

Admissible mean dynamics possess important features: there is a unique solution trajectory $x : \mathbb{R}_+ \to X$ from any initial point $x \in X$, all solution trajectories stay in the space $X$, all rest points of $V$ are Nash equilibria of $F$, and all accumulation points of solution trajectory $x$ are critical points of $L \circ x$. If in addition, $\nabla L(x)^t V(x) = 0$ implies $V(x) = 0$, then $V$ globally converges to a Nash equilibrium of $F$.

It is clear that $x$ is a Nash equilibrium of the traffic congestion game if it solves $D_r(x_r) - v_r t(x) = 0$ for all $r \in R$. The following example shows an admissible mean dynamic for this game.

**Example 1.** Suppose the marginal agent of population $r$ drives whenever he observes a positive payoff of driving. This mean dynamic is characterized by $V_r(x) = v_r F_r(x)$ for all $r \in R$, is admissible and globally converges to the unique Nash equilibrium of the game. Let $L(x) = \frac{1}{2} \sum_R F_r(x)^2$. We show that $L$ is a Lyapunov function for $V$. $\frac{\partial L}{\partial x_s} = \sum_R F_r(x)(\frac{\partial D_r}{\partial x_s} - v_r t'(x)) = F_s(x) D'_s(x_s) - t'(x) \sum_R v_r F_r(x)$ for all $s \in R$. $\nabla L(x)^t V(x) = \sum_s v_s F_s(x)^2 D'_s(x_s) - t'(x)(\sum_R v_r F_r(x))^2 < 0$ whenever there is $s$ such that $F_s(x) \neq 0$ and $\nabla L(x)^t V(x) = 0$ if and only if $x$ is the Nash equilibrium.
2 Traffic congestion system

The aggregate welfare for a strategy distribution $x$ is given by $w(x) = \sum_{R} [\int_{0}^{\infty} D_r(z)dz - v_r t(x) x_r]$. A social planner would select a strategy distribution that maximizes welfare. The optimal strategy distribution $x^*$ is characterized by the first order conditions of $f$, $\frac{\partial f}{\partial x_s} = D_s(x^*_s) - v_s t(x^*) - t'(x^*) \sum_{R} v_r x^*_r = 0$ for all $s \in R$. In real life, there are no social planners but policy makers facing informational and political constraints. In the following sections we analyze how a policy maker could implement or approximate the social planner’s solution under different informational and political constraints.

In real life, $t$ is observable and hence we assume that policy makers know $t$. In addition, we assume the policy maker knows the support of $v_r$, i.e. the possible values of time are known, but not the distribution.

2.1 Full information

Suppose a policy maker had complete information about the commuting time function $t$, the demand functions $D_r$, and the values of time $v_r$ for all $r \in R$, then he could calculate the optimal allocation $x^*$ and impose a fixed optimal tax $T^* = t'(x^*) \sum_{R} v_r x^*_r$. From the point of view of a driver of type $r$, now payoffs have been modified and are characterized by $F^T_r(x) = D_r(x_r) - v_r t(x) - T^*$.

Example 2. The mean dynamic defined by $V_r(x) = v_r F^T_r(x)$ for all $r \in R$ globally converges to the social optimum.

2.2 Unknown demand

In this section we assume the policy maker has no information regarding the demand, i.e. $D_r$ is unknown for all $r \in R$. This lack of information prevents the policy maker from implementing the optimal fixed tax $T^* = t'(x^*) \sum_{R} v_r x^*_r$. 

Suppose the policy maker could distinguish the type of every driver on the road; in this case a
dynamic tax \( T^D(x) = t'(x) \sum_R v_r x_r \) could be implemented. In this section we assume the policy maker
can do so. We consider this to be a very strong assumption, but include it because it allows to study
the gradual effects of losing information from the policy maker’s perspective.

When the dynamic tax is imposed, type \( s \) observes a payoff equal to \( F^D_s(x) = D_s(x_s) - v_s t(x) - t'(x) \sum_R v_r x_r \) and the conditions for Nash equilibrium in this game are identical to the conditions for a
social optimum.

Despite the striking notational similarity between this case and the previous one, these two games
possess very different evolutionary properties. The dynamic tax game is both a potential and a stable
game. The fixed tax game is neither potential nor stable.

A game \( F \) is potential if there is a function \( f \) such that \( \nabla f = F \). Potential games have very strong
convergent properties.

**Proposition 1.** Let \( F \) be a potential game, and let \( V \) be a dynamic which is admissible with respect
to \( F \). Then every solution trajectory of \( V \) converges to a connected set of Nash equilibria of \( F \).

In our case, \( F^D \) is a potential game (\( \nabla w = F \)) with a unique Nash equilibrium, hence every admissible
mean dynamic \( V \) will converge to it. In this case the fifth condition of admissibility becomes very
intuitive i.e. \( V(x) F^D(x) \geq 0 \) i.e. at every moment the population is moving towards higher payoffs.

\( F^T \) is not potential since \( \frac{\partial F^T}{x_r} \neq \frac{\partial F^T}{x_s} \) for all \( s \neq r \).

Before we introduce the definition of stable game we define the concept of evolutionary stability. A
state \( x \) is globally evolutionary stable if \( (y - x)'F(y) < 0 \) for all \( y \in X \setminus \{x\} \). Intuitively, \( x \) is evolutionary
stable if regardless of the current state, the average payoff would increase by moving towards \( x \). A
game \( F \) is stable if \( (y - x)'(F(y) - F(x)) \leq 0 \) for all \( x, y \in X \).

**Proposition 2.** Suppose \( F \) is a strictly stable game and let \( x \) be a Nash equilibrium of \( F \), then \( x \) is
evolutionary stable.

In our case, \( F^D \) is a strictly stable game because \( f \) is a strictly concave function. \( F^T \) is not stable
since \( \sum_R (y_r - x_r)(F^T(y_r) - F^T(x_r)) = \sum_R D'_r(c_r)(y_r - x_r)^2 - t'(x - y) \sum_R v_r(y_r - x_r) > 0 \) for some \( y \)
and \( x \) in \( X \).
2.3 Unknown demand and unknown social cost

In this section we assume the policy maker has no information regarding the demand or social cost other than the support of the value of time. Policy makers can observe the total number of drivers on the road, but cannot distinguish their types. Thus the implementation of the dynamic tax $T^D(x) = t'(x)\sum_{R} v_r x_r$ becomes impossible.

Suppose all drivers had an innate tendency for telling the truth when asked about their private information. In this case, the policy maker would only need to ask drivers to report their value of time and value of commuting to calculate the social optimum distribution $x^*$ and the socially optimal fixed tax. We do not believe this is a suitable solution. It was argued above that dynamic taxes have better properties than fixed taxes; hence a dynamic tax would be a better solution. Using a dynamic tax would, in addition, reduce the amount of information to be reported. It would only be necessary to ask drivers to report their value of time.

Suppose the policy maker asks drivers to report their value of time and that $x_i$ drivers report having $v_i$ as their value of time. With this information, the message tax $T^M(x) = t'(x)\sum_{R} v_i x_i$ can be imposed. Notice that for this section, the strategy space is increased to incorporate the messages. In previous sections $x_r$ represents the amount of drivers on the road of type $r$ (a scalar). In this section, $x_r = (x_{r1}, \ldots, x_{rR})$ is a vector and $x_{ri}$ is a amount of drivers of type $r$ who send the value $v_i$. To simplify notation, $x_r$ also represent the total amount of drivers of type $r$.

The payoffs associated with this message tax are defined by $F^M_{si}(x) = D_s(x_s) - v_s t(x) - t'(x)\sum_{R} v_r x_r$. Nash equilibria are characterized by $F^M_{si}(x) = D_s(x_s) - v_s t(x) - t'(x)\sum_{R} v_r x_r = 0$ for all $(s, i)$ in $R \times R$.

This game possess a continuum of equilibria characterized by the distribution of messages sent. Let $z(x) = \frac{\sum_{R} v_r x_r}{x}$ be the average message sent. Clearly, $z$ ranges from $v_1$ and $v_R$, the lowest and highest values of time. For every $z \in [v_1, v_R]$ there is a Nash equilibrium satisfying $F^M_{si}(x) = D_s(x_s) - v_s t(x) - t'(x)zx = 0$ for all $(s, i)$ in $R \times R$.

Despite having a plethora of equilibria, not being potential nor stable; the success of $T^M$ completely depends on the aggregate behavior of drivers. For example, suppose drivers always tell the truth. In this case, the social optimum would be the observed equilibrium. More generally, we say that an admissible mean dynamic $V$ is hopeful if it is also admissible for $F^D$. 

Example 3. The mean dynamic $V$ is average truth-telling if and only if $\dot{x} > 0 \iff T^M(x) > T^D(x)$ and $\dot{x} < 0 \iff T^M(x) < T^D(x)$. Since $V(x)'F^D(x) = V(x)'F^M(x) + \dot{x}\sum_{r,i}(v_i - v_r)x_{ri} \geq 0$, an average truth-telling is hopeful.

2.4 Revenue neutrality

On top of the informational constraints, policy makers usually face political constraints. In the case of externality abatement, the imposition of a new tax is usually seen as a bad alternative, since it involves a flow of resources from the population to the the government. Hence we consider important to offer revenue neutral alternatives.

In the context of our model, revenue neutrality is simple to achieve since any tax can be replaced by a smaller tax on driving and a subsidy on not driving. For example, the dynamic tax $T^D(x) = t'(x)\sum_R v_rx_r$ can be replaced by a smaller tax $T^{BD}(x) = \frac{m-x}{m}t'(x)\sum_R v_rx_r$ and a subsidy $S^{BD}(x) = \frac{x}{m}t'(x)\sum_R v_rx_r$. The analogous division can be implemented for the message tax.

It is worth noting that achieving revenue neutrality is difficult or impossible when more demanding solution concepts are used. In particular, dominant strategy implementation makes achieving revenue neutrality impossible in this model.

3 Experiment

The main objective of the experiment is to test if the message system proposed above allows drivers to converge to the socially optimal traffic congestion level. The previous section provides some evidence that, under admissibility of the behavioral rules, the social optimum would be observed. We observe and measure the real effectiveness of the system in the experimental laboratory.

Our experimental design consists of a driving game in which 14 subjects independently decide whether to “drive” or “not drive” on a fixed road for 30 rounds of play.\footnote{In two out of six session the number of drivers was 16.} At the beginning of each game, every subject was randomly and privately assigned a type given by two numbers: a value of commuting and a value of time. These values are held fixed over the 30 rounds of play. Neither
the distribution nor the support of values was revealed to the subjects. Actual types were chosen to simulate a large number of subjects and represent a complex distribution of values. There is a fixed set of types, described below.

Congestion occurs when thousands of drivers use the road at the same time. However, designing an experiment that requires thousands of subjects would be both impractical and expensive. We address the large numbers problem through our experimental design. When there is a large number of drivers, the impact each individual has on one another is small. We include this characteristic in our experiment with a small number of drivers by controlling the impact that the actions of one driver have on the incentives of other drivers. To illustrate this feature, suppose in a given moment 6 subjects are driving and 8 are not. This combination is not necessarily rational i.e. it is possible that some of the subjects who are driving would be better off by not driving and vice versa. Suppose one subject who is currently driving stops driving. This action has an impact on all others drivers’ payoffs because it affects commuting times and possibly the tax. However, we design our experiment so that this action changes the optimality (or non-optimality) of driving for at most one other driver i.e. in our experiment there is always at most one marginal driver.

The particular set of values assigned to different types was also chosen to minimize the set of drivers who belong to both the optimal allocation and the Nash equilibrium when there is no tax (the inefficient equilibrium). The inefficient equilibrium consists of 10 drivers and the optimal allocation consists of 6. However, only two drivers belong to both allocations, meaning that 12 out of 14 drivers have to change their behavior with the introduction of the system. We believe that such a radical change in the allocation is a strong test for the effectiveness of the system.

Figures 1 and 2 contain the list of types used in the experiment and illustrate their distribution. The congestion function $t(x) = \frac{x^3}{12}$ was chosen to have commute values and time values on a relatively equal scale.

In figure 2 every dot represents a type; the red line represents the equilibrium time when there is no tax and the blue line represents the optimal time when the optimal tax is imposed. The gray lines are variations of time when a driver is added or substracted.

In theory, with the above types and congestion function, the inefficient equilibrium achieves an efficiency level of 301.3 experimental dollars. Whereas the optimal allocation achieves an efficiency of
406.3 experimental dollars, an increase of 34.8%. Several experiments,\textsuperscript{8} however, have shown that the inefficient equilibrium is usually not observed. For this reason, we measure the effectiveness of the traffic congestion system based on observed efficiencies.

In order to assess the general effectiveness of the system and the relative costs of losing information from the policy maker’s perspective we considered 6 treatments: i) \textit{No tax} - the benchmark to measure the level of congestion; ii) \textit{Fixed tax} - the optimal fixed tax assuming the policy maker knows the demand and marginal social cost; iii) \textit{Dynamic tax} - it is assumed that the policy maker knows the marginal social cost, but not the demand; iv) \textit{Message tax} - this treatment represents the system we are proposing, it assumes that the policy maker does not know the demand or the marginal social cost. The last two treatments are identical to the dynamic and message treatments with the additional restriction that the system needs to be revenue neutral.

A traffic congestion system is effective if and only if it increases efficiency. Measuring efficiency gains with a discrete and small number of drivers, however, could be difficult as the range of possible outcomes (and sessions) is small. In order to increase our precision we increase the number of drivers in the experiment by letting every experimental subject manage ten identical drivers. In every round, each subject decides whether to drive or not; if he decides to drive, a driver of his type is introduced to the road (up to ten); if he decides to not drive, a driver is removed from the road (up to zero).

The experiment was run at the Experimental Economics Lab at the University of Maryland. There

\textsuperscript{8}See Rapoport et al. [2004], Anderson et al. [2007], and Rapoport et al. [2014] for examples.
were 88 participants, all undergraduate students at the University of Maryland. There were six sessions. No subject participated in more than one session. In every session, subjects participated in six different treatments. Treatments were played in random order. Participants were seated in isolated booths. The experiment is programmed in z-Tree (Fischbacher [2007]).

At the beginning of each treatment, each subject was randomly assigned a type, i.e. a value of commuting \( D \) and a value of time \( v \). In addition, they were informed that in some rounds they could face a tax \( T \) or a subsidy \( S \) and that their experimental payoffs would depend on the observed time \( t \) using the following formulas: \( D - \frac{vt}{60} - T \) for driving and \( S \) for not driving. In all rounds, subjects could see on screen the current values of \( T \) and \( S \), the history of times for all previous rounds and their private information. In addition, a table with several time scenarios (\( t = 5 \) to \( t = 85 \) in steps of 5) with the values for driving and not driving was provided.

Subjects were informed that in some sections (treatments) they could be asked for their value of time and were instructed to “send one of the available messages”. Subjects were informed that messages would be used to calculate taxes and subsidies for the next period, but the exact mechanism was not explained.

Subjects were explained in detail how earnings were calculated. In every round \( r \), subjects received \( x_r = (0.9764)^{30-r} \ (x_{30} = 1, x_1 = 0.5) \) “points” for a conditionally optimal action and 0 otherwise. This payment scheme fulfills two purposes. First, no Nash equilibrium is favored; remember that for the message treatment there are many equilibria for this game. Second, it provides incentives for agents to adjust their strategies over time. Dollar earning were calculated by adding up all points and multiplying this quantity by 0.107675. This constant was calculated, and explained as such, to produce a range from $0 to $14 dollars. In addition, subjects were paid a $6 show up fee. Subjects received an average payment of $18.28.

The following section present the results of the experiment and gives a general description of some stylized facts.
3.1 Experimental results

We present in this section the results of the experiment. For every treatment, we include 3 figures depicting the evolution of the behavior in three different dimensions: number of drivers on the road, efficiency and types.

In the first two types of figures, the solid blue line represents the average across sessions and the red line is the average across sessions and periods at least 11.

The third kind of figure shows the frequency of driving periods by type of driver. In this figure, there are two gray lines representing the efficient allocation and the Nash equilibrium. The green line is the observed average.

3.1.1 No tax

This treatment establishes the reference point to assess the gains attained by the tax treatments. The observed behavior is as follows.

![Figure 3: Number of drivers](image)

![Figure 4: Efficiency](image)

The Nash equilibrium is 10 drivers and the observed average is in line with this number. Efficiency, on the other hand, converges to a lower number. This is due to the fact that some drivers who do not drive on equilibrium were trying to drive. This reduces efficiency because these drivers get negative payoffs ($0 dollars for that round) and increase the time experienced for others. Convergence in types can be observed in the following figure.
3.1.2 Fixed Tax

This treatment establishes the possible efficiency achievable by tax treatments. In theory, the socially optimal allocation could be expected. In practice, the observed outcome depends on the behavior of drivers. The observed behavior is as follows.

The Nash equilibrium is 6 drivers and the observed average is in line with this number. Efficiency also converges to the socially optimum equilibria. Convergence in types can be observed in the following figure.

It can be observed that the reduced efficiency is due to type 12 fails to drive 100% of the time, despite having incentives to do so.

3.1.3 Dynamic Tax

In theory, this treatment possesses the best convergent properties of all the treatments. As expected, the social equilibrium is observed.
Convergence to equilibrium in identity of drivers, on the other hand was not observed for all types. The following figure shows that type 12 fails to drive 100% of the time. Furthermore, this failure is exploited by type 10.

3.1.4 Message Tax

In previous treatments it was assumed that the policy maker had some information regarding the demand or marginal social cost. In this treatment it is assumed that the policy maker has none.
Taxes depend completely on messages sent by subjects. Not very surprisingly, this treatment did not converge to the social optimum. However, very surprisingly, it did converge and the point of convergence produces a high level of efficiency.

![Figure 12: Number of drivers](image1)

![Figure 13: Efficiency](image2)

Convergence to the socially optimum identity of drivers was also not observed. The following figure shows that type 12 fails to drive 100% of the time, strategy that would guarantee a positive payoff. This failure is exploited by types 10 and 9.

![Figure 14: Types](image3)

### 3.1.5 Balanced Dynamic Tax

Many times policy makers face political constraints to impose new taxes, even if they are corrective. This treatment tests if achieving revenue neutrality is possible (together with a high level of efficiency). As seen in the following figures, revenue neutrality is possible.
3.1.6 Balanced Message Tax

Revenue neutrality (together with a high level of efficiency) is possible, even when taxes and subsidies depend on messages.
3.2 Analysis

The main objective of the experiment is to test if the message system allows drivers to converge to the socially optimum traffic congestion level. We first focus on efficiency and the number of drivers.

Table 1 contains the mean efficiency achieved in every treatment as a percentage of the mean efficiency obtained by the fixed tax treatment. The standard deviation has been scaled accordingly. The sub-table in the middle contains p values for the hypothesis that the row treatment and the column treatment have the same efficiency against the alternative that the row has a higher efficiency. A paired Wilcoxon signed-Rank test was used instead of the more common Mann-Whitney-Wilcoxon test in case a session effect or period effect was present. The lower portion of the table shows the results for the number of drivers on the road. Estimates have not been scaled because units represent subjects decisions directly. P values are also reported for the number of drivers. The alternative hypothesis is that the row treatment has a lower number of drivers than the column treatment. The last column represents the largest deviation from the mean for all periods considered. The minimum possible deviation for the number of drivers is 0.1. All estimates are calculated considering only period at least 11.

Figures 21 and 22 show estimates for efficiency and the number of drivers for different choices of initial period of analysis. All treatments are significantly (p values < 0.0001 for all periods of analysis) more efficient than the no-tax treatments. Dynamic treatments and the fixed treatment achieve a significantly (p values < 0.0001 for all periods of analysis) higher efficiency than message treatments.
### Table 1: Estimates for Period $\geq 11$

<table>
<thead>
<tr>
<th>Period</th>
<th>Treatment</th>
<th>Measure</th>
<th>Mean</th>
<th>SD</th>
<th>No Tax</th>
<th>Fixed</th>
<th>Dynamic</th>
<th>Message</th>
<th>Bdynamic</th>
<th>Bmessage</th>
<th>Epsilon</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>No Tax</td>
<td>efficiency</td>
<td>65.91%</td>
<td>13.01%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>24.34%</td>
</tr>
<tr>
<td>11</td>
<td>Fixed</td>
<td>efficiency</td>
<td>100.00%</td>
<td>4.12%</td>
<td>9.97E-22</td>
<td>3.90E-01</td>
<td>6.52E-20</td>
<td>8.89E-01</td>
<td>4.66E-20</td>
<td>9.90E-01</td>
<td>3.58%</td>
</tr>
<tr>
<td>11</td>
<td>Dynamic</td>
<td>efficiency</td>
<td>100.53%</td>
<td>3.26%</td>
<td>9.97E-22</td>
<td>1.01E-02</td>
<td>1.12E-01</td>
<td>1.10E-21</td>
<td>8.89E-01</td>
<td>4.66E-20</td>
<td>3.28%</td>
</tr>
<tr>
<td>11</td>
<td>Message</td>
<td>efficiency</td>
<td>95.00%</td>
<td>3.44%</td>
<td>9.98E-22</td>
<td>3.41E-01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.51%</td>
</tr>
<tr>
<td>11</td>
<td>Bdynamic</td>
<td>efficiency</td>
<td>101.05%</td>
<td>2.85%</td>
<td>9.94E-22</td>
<td>1.01E-02</td>
<td>1.12E-01</td>
<td>1.10E-21</td>
<td>2.14E-16</td>
<td></td>
<td>3.28%</td>
</tr>
<tr>
<td>11</td>
<td>Bmessage</td>
<td>efficiency</td>
<td>94.35%</td>
<td>4.86%</td>
<td>9.98E-22</td>
<td>6.60E-01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.32%</td>
</tr>
</tbody>
</table>

Figure 21: Efficiency estimates

Figure 22: No. of Drivers estimates

95% confidence interval are shown in Figures 23 and 24

Figure 23: Efficiency estimates

Figure 24: No. of Drivers estimates
3.2.1 Message Tax

In this section we document several regularities that allowed the message system to achieve near-optimum levels of efficiency. First, reported average values of time are lower than the true averages. Second, reported average values of time are at least half of the true averages. Third, the dynamic system allows subjects to better coordinate at equilibrium.

Figure 25 shows the average message sent by type. Blue bars represent the set of optimal drivers. Figure 26 shows the number of times a particular message was received by the system as a proportion of the total number of messages received.

Taxes depend on messages sent by subjects currently on the road and not on all messages received by the system i.e. taxes depend on the average message \( z(x) = \frac{\sum x_r}{R} \). Figure 27 shows the evolution of the average message and the evolution of what the average message would be if all subjects had sent their true value of time.

Let \( G \) be the distribution of actual messages sent to the system, i.e. \( z \sim G \). Figures and show the empirical density and distribution of \( z \).
Figure 30 shows the relationship between average real messages and average sent messages. A Wilcoxon test was used to reject the null hypothesis that the difference between the two types of messages is symmetric around zero versus the alternative that the average real messages are greater. The test has a p-value < $2.2E^{-16}$. The same test was used to accept the hypothesis that sent messages are greater than one half the real average. The test has a p-value < $2.2E^{-16}$. These bounds help explain the efficiency levels in the message treatment.

Let $z^*$ be the equilibrium average message when all drivers send their true value of time. Since average sent messages are smaller than average real messages we have that $G(z^*) = 1$ i.e. the highest observed average message will always be below the real equilibrium average message. Let $f(z)$ be the achieved efficiency when $z$ is sent to the system. Then, unless $G$ is degenerate,
$E[f(z)] > f(0)$ i.e. the implementation of the message system is guaranteed to generate efficiency gains, unlike policy guesses about the value of time. As an example, suppose $G$ is uniform, then the minimum efficiency of the message system would be $\frac{1}{2} + \frac{f(0)}{2}$. Figure 31 compares the minimum efficiency for three scenarios against a policy guess.

![Figure 31: Nash Equilibria](image)

In the experiment, $G$ was not uniform. Figure 32 shows the previous figure for the experimental design. The solid blue line represents the efficiency achieved in different Nash equilibria as a percentage of the efficiency achieved by the fixed optimal tax. The yellow dots represent their experimentally observed counterparts.

**References**


Figure 32: Equilibria in the Message Treatment


Xiaolei Guo and Hai Yang. Pareto-improving congestion pricing and revenue refunding with multiple


Amnon Rapoport, William E. Stein, James E. Parco, and Darryl A. Seale. Equilibrium play in single-server queues with endogenously determined arrival times. *Journal of Economic Behavior & Or-


INSTRUCTIONS

Thank you for participating in today’s experiment. This is an experiment in the economics of decision-making. Various research foundations have provided funds for this research. The instructions are simple. If you follow them carefully and make good decisions, you will earn money.

The entire experiment should be complete within ninety minutes. You will be paid a $3 show-up for showing up on time. In addition, you will be paid $3 if you complete the session. You may collect the show-up fee at any moment if you decide to terminate your participation. If you complete the session, you will be privately paid both fees at the end. In addition, you will be paid a performance fee depending on your decisions. The mechanics for this payment are detailed below. All quantities in the experiment are measured in experimental units (EU). The connection between experimental units and real dollars is explained below.

This experiment is organized in 6 sections. Every section is divided into 30 rounds. In each round, you and other participants will decide whether to “drive” or “not drive” on a fixed road. The length of your commute will depend on the number of drivers on the road. At the beginning of each section, you will be assigned two numbers. These numbers are your private information.

1. Your value of commuting
2. The value of your time

In every section, you may or may not face a tax to use the road. In addition, a subsidy could be offered not to drive. You will be informed whether or not a tax will be charged or a subsidy offered before you make your decision every round. In some sections, you will be able to affect the value of the tax or subsidy by sending a message regarding the value of your time.

At the end of every round you will be informed of the following elements:

i. The time spent on the road
ii. Your payoff (in experimental units)

Your payoff will depend on your decisions, your values, the time spent on the road and the tax.

Not driving guarantees a payoff (in experimental units) of S, where S is the value (possibly 0) of the subsidy offered not to drive.
Driving has a payoff (in experimental units) equal to $U - \frac{V}{60}t - T$, where $U$ is your value of commuting, $V$ is your value of time, $T$ is a tax (possibly 0) to use the road and $t$ is the time spent on the road.

**Performance fee**

All six sections have the same performance fee structure. In every round you will be faced with two decisions: to drive or not to drive.

Depending on your values, the time, the tax and the subsidy, driving could be better, equal or worse than not driving. Your payment in each round will only depend on the optimality of your decision. If you select the option with the higher payoff in round $r$, you will earn $x_r = (0.9764)^{30-r}$ experimental units; otherwise you will earn $0$.

Your performance fee will be then calculated as the sum of your profit in every round multiplied by a factor of $0.107675$.

**Example**

Suppose that your value of commute $U$ is equal to 20 and your value of time $V$ is equal to 2. In round 1 the time spent on the road is 10, there is a tax of 3 and a subsidy not to drive of 1. Then:

Not driving payoff: 1

Driving payoff: $20 - \frac{2}{60} \times 10 - 3 = 16.666$

In this case, the better option is to drive. If you decide not to drive you will earn 0 EU. If you decide to drive you will earn $x_r = (0.9764)^{30-1} = 0.5$.

**Questions**

If you have any questions right now, please share it with all.

If you have any question during the experiment, please quietly raise your hand and one of the experimenters will come to you to answer your question. It is important that you do not talk with any of the other participants.