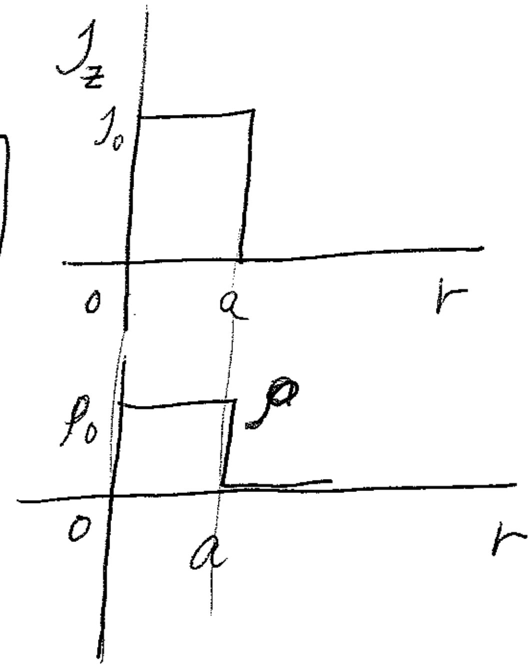
Ideal Kink Mode in Cylinder

$$\boxed{\begin{aligned} \cancel{\gamma}\,\vec\nabla_\perp\cdot(\rho\vec\nabla_\perp\tilde\varphi) &= \widetilde{\vec B\cdot\vec\nabla J_z} \\ J_z &= \nabla_\perp^2\psi \\ \cancel{\gamma}\,\tilde\psi &= \vec B\cdot\vec\nabla\tilde\varphi \end{aligned}}$$



↑ in plasma

$$\vec B = B_0\hat z + \hat z\times\vec\nabla_\perp\psi$$

$$\vec u \equiv \hat z\times\vec\nabla\varphi$$

In vacuum $\quad J_z = \nabla_\perp^2\psi = 0$

The first plasma equation also applies to vacuum since the RHS is just the Maxwell stress tensor ($E \ll B$) and the LHS is plasma momentum which $\to 0$ in vacuum. EM momentum is small since $E \ll B$.

<u>Vacuum</u> $\quad \nabla_\perp^2 \breve{\psi} = 0 \quad , \quad \breve{\psi} \to e^{im\theta} e^{-ikz} e^{\gamma t}$

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{d\breve{\psi}}{dr}\right) - \frac{m^2}{r^2}\breve{\psi} = 0$$

$$\Rightarrow \boxed{\breve{\psi}_{vac} = \left(\frac{r}{a}\right)^{-m}}$$

<u>Plasma</u>

$$\vec{B}\cdot\vec{\partial} \to -iB_0 k + \frac{im}{r}B_\theta \quad , \quad k \to \frac{n}{R}$$

$$\equiv -ik_\parallel B_0$$

$$\boxed{k_\parallel \equiv (n - m/q)/R = (nq - m)/qR}$$

For given ρ and J_z profiles, $k_\parallel = \text{const}$

$$\Rightarrow \gamma\rho_0 \nabla_\perp^2 \breve{\varphi} = -ik_\parallel \nabla_\perp^2 \breve{\psi} B_0 \qquad r \neq a$$

$$\gamma\breve{\psi} = -ik_\parallel B_0 \breve{\varphi}$$

$$\Rightarrow \tilde{\varphi} = \gamma\tilde{\psi}/(-ik_{\parallel}B_0), \quad \boxed{k_{\parallel} \neq 0}$$

assume $k_{\parallel} \neq 0$ in $r<a$

$$\Rightarrow \left(\gamma^2 + k_{\parallel}^2 B_0^2/\rho_0\right) \nabla_{\perp}^2 \tilde{\psi} = 0$$

$$\boxed{\gamma^2 \neq -k_{\parallel}^2 V_A^2} \Rightarrow \nabla_{\perp}^2 \tilde{\psi} = 0$$

$$\Rightarrow \boxed{\tilde{\psi}_{pl} = \left(\frac{r}{a}\right)^m}$$

satisfies
$\tilde{\psi}_{pl} = \tilde{\psi}_{vac}$ at $r=a$

must be the case if $[\tilde{B}_r] = 0 = [\partial_\theta \tilde{\psi}]$

Momentum jump condition

$$\Rightarrow \gamma[\rho\tilde{\varphi}'] = -ik_{\parallel}B_0[\tilde{\psi}'] - \frac{im}{r}[J\tilde{\psi}]$$

$$\Rightarrow -\gamma\rho_0\tilde{\varphi}'_{pl} \simeq +ik_{\parallel}B_0\frac{(2m)}{a} + \frac{im}{r}J_0$$

$$\tilde{\varphi}' \Rightarrow \frac{\gamma\tilde{\psi}'}{-ik_{\parallel}B_0} = \frac{\gamma m}{-ik_{\parallel}B_0 a}$$

$$\Rightarrow \boxed{\frac{\omega^2}{2V_A^2} = k_{\parallel}^2 + \frac{k_{\parallel}}{qR}}$$

all @ $r = a$

$$\boxed{\frac{\omega^2 q_a^2 R^2}{2V_A^2} = (m - nq_a)\left[(m - nq_a) - 1\right]}$$

$k_{\parallel} \neq 0$ for $r < a$