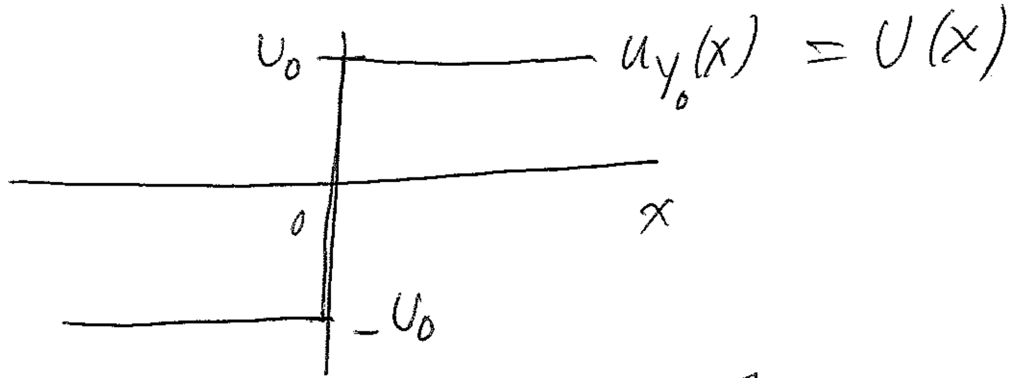
Kelvin-Helmholtz - Step fn profile



let $\boxed{\bar{\omega} = \omega - kU}$, $e^{iky} e^{-i\omega t}$

eV eqn: $\bar{\omega} D^2 \tilde{\varphi} = -kU'' \tilde{\varphi}$

$$\Rightarrow \boxed{\bar{\omega} D^2 \tilde{\varphi} = \bar{\omega}'' \tilde{\varphi}}$$

$$x \neq 0 \Rightarrow U'' = 0 \Rightarrow D^2 \tilde{\varphi} = 0$$

$$\Rightarrow \text{let } \tilde{\varphi}(x) = \begin{cases} A_+ e^{-kx}, & 0 < x \\ A_- e^{+kx}, & x < 0 \end{cases}$$

Jump conditions:

$$x \to 0 \Rightarrow \bar{\omega} \tilde{\varphi}'' \simeq \bar{\omega}'' \tilde{\varphi}$$

$$\Rightarrow (\bar{\omega} \tilde{\varphi}')' \simeq (\bar{\omega}' \tilde{\varphi})'$$

$$\int_{-\epsilon}^{+\epsilon} dx \Rightarrow \boxed{[\bar{\omega} \tilde{\varphi}']_-^+ = [\bar{\omega}' \tilde{\varphi}]_-^+} \quad \text{J1}$$

integrate once $\Rightarrow$

$$\bar{\omega}\hat{\varphi}' = \bar{\omega}'\hat{\varphi} + c$$

$$\Rightarrow \frac{\hat{\varphi}'}{\bar{\omega}} = \frac{\bar{\omega}'\hat{\varphi}}{\bar{\omega}^2} + \frac{c}{\bar{\omega}^2}$$

$$\Rightarrow \left(\frac{\hat{\varphi}}{\bar{\omega}}\right)' = \frac{c}{\bar{\omega}^2} \overset{\int dx}{\Rightarrow} \boxed{\left[\frac{\hat{\varphi}}{\bar{\omega}}\right]_-^+ = 0} \quad J2$$

J1 & J2 are 2 jump condns.

Note $\bar{\omega}'(\pm\epsilon) = 0$

$$\therefore J1 \longrightarrow \boxed{\left[\bar{\omega}\hat{\varphi}'\right]_-^+ = 0} \quad J1'$$

Apply J2 $\Rightarrow$ $\boxed{\dfrac{A_+}{\bar{\omega}_+} = \dfrac{A_-}{\bar{\omega}_-}}$

Apply J1' $\Rightarrow$ $-k\bar{\omega}_+ A_+ = k\bar{\omega}_- A_-$

$$\Rightarrow \boxed{\bar{\omega}_+ A_+ = -\bar{\omega}_- A_-}$$

$$\Rightarrow \bar{\omega}_+^2 = -\bar{\omega}_-^2 \Rightarrow \bar{\omega}_+ = \pm i\bar{\omega}_-$$

$$\Rightarrow \omega - kU_0 = \pm i(\omega + kU_0)$$

$$\Rightarrow \boxed{\omega = \pm ikU_0} \quad \text{Always unstable}$$

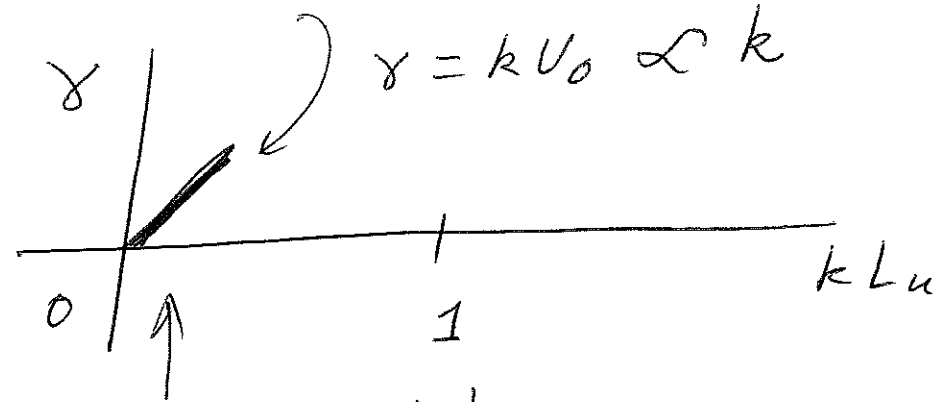
Sharp boundary $\Rightarrow L_u \to 0$

$\Rightarrow$ above good $\Leftrightarrow kL_u \ll 1$

<u>and</u> $L_u \ll (\partial/\partial x)^{-1}$

$\Rightarrow kL_u \ll 1$

$\therefore$



only good here

Need gentler $U_{y0}(x)$ to do $kL_u \gtrsim 1$.

<u>Note</u> $\bar{\omega}_+ = +i\bar{\omega}_- \Rightarrow iA_+ = -A_-$

$$\Rightarrow \boxed{A_+ = iA_-}$$

$$\Rightarrow \tilde{\varphi} = A_- \begin{cases} i e^{-kx} e^{iky}, & 0 < x \\ e^{kx} e^{iky}, & x < 0 \end{cases}$$

$$Re(\tilde{\varphi}) \to \begin{cases} -e^{-kx} \sin ky, & 0 < x \\ e^{kx} \cos ky, & x < 0 \end{cases}$$