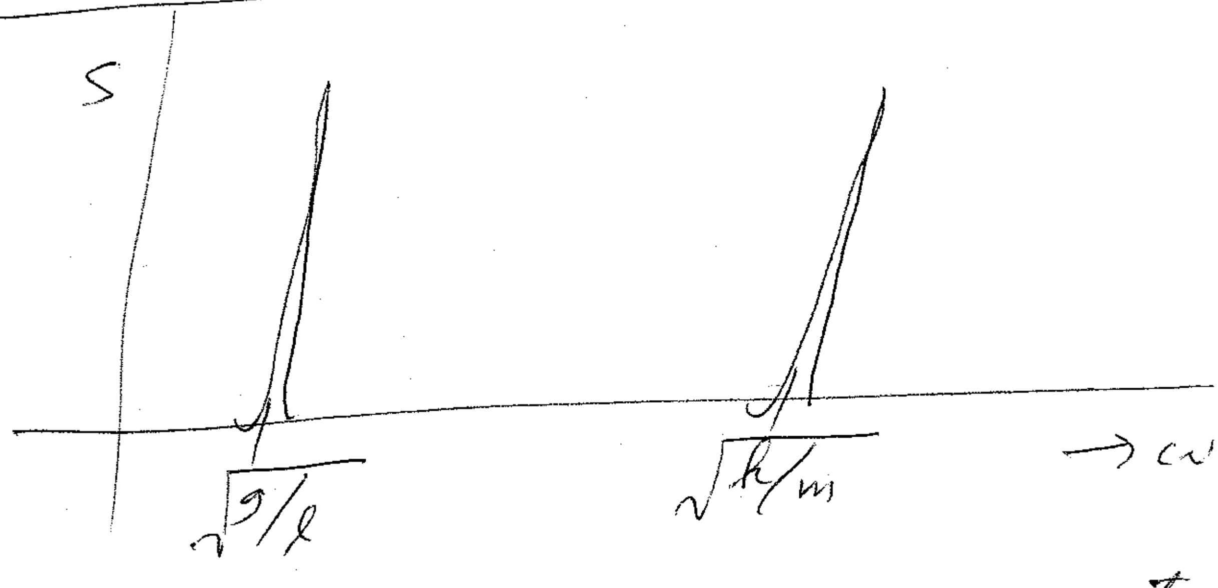
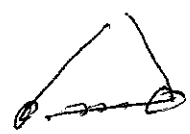
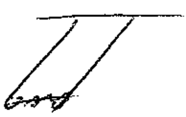
Frequency scale for normal modes and excitation thereof



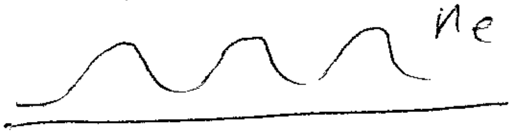
Spectral responses are disparate.

- To excite high frequencies, need compression in i.c.





- To excite low frequency, need incompressible displacement

Likewise, 

$\Rightarrow$ high frequency, $\omega \approx \omega_p$

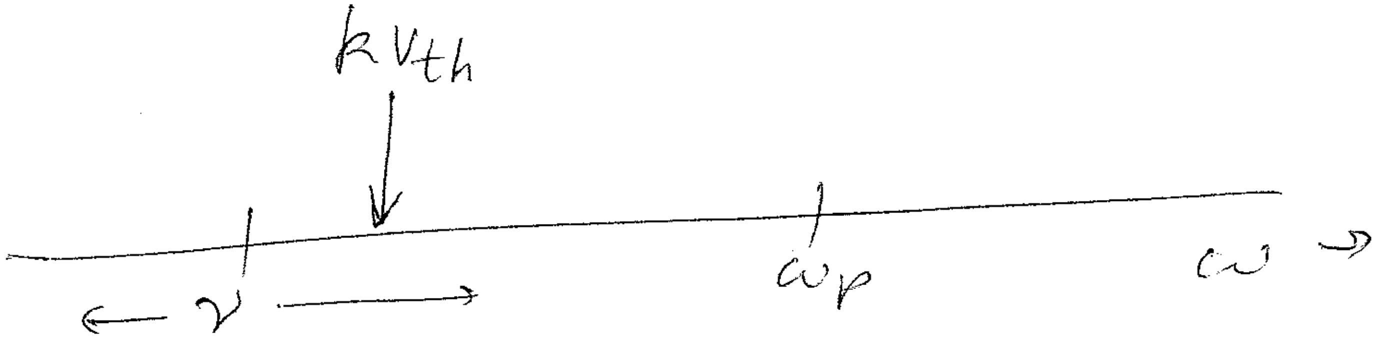
$(k\lambda_D \ll 1)$

assume

$\Rightarrow$ low frequency

High Frequency Plasma Response

$k\lambda_D \ll 1 \Rightarrow kv_{th} \ll \omega_p$



Consider $\omega \sim \omega_p \gg kv_{th}$ — (A)
(sloppy on species)

Then, also, $\omega \gg \nu$ — (B)

(B) $\Rightarrow$ Neglect $C(f,f)$

Look at Moment equations

$$\partial_t n_\alpha + \partial_x (n_\alpha u_\alpha) = 0$$

$$\partial_t (n_\alpha M_\alpha u_\alpha) + \partial_x (n_\alpha M_\alpha u_\alpha u_\alpha) = -\nabla \cdot \overleftrightarrow{P}_\alpha + n_\alpha e_\alpha (\vec{E} + \vec{u}_\alpha \times \vec{B})$$

small

Compare $\partial_t (nMu) : \nabla \cdot P$

$\sim \omega n M u : \frac{nMv_{th}^2}{L} \sim \omega^2 : \frac{v_{th}^2}{L^2}$, if $\omega \sim \frac{u}{L}$

Thus, $\omega >> k v_{th}$

$\Rightarrow \vec{\nabla} \cdot \overleftrightarrow{P}$ can be thrown away

$\Rightarrow$ Cold (collisionless) plasma fluid

$$\boxed{\begin{aligned} &\partial_t n_\alpha + \partial_x (nu)_\alpha = 0 \\ &\partial_t (nMu)_\alpha + \partial_x (nMuu)_\alpha = \\ &\quad (ne)_\alpha \left[\vec{E} + \vec{u}_\alpha \times \vec{B} \right] \end{aligned}}$$

Closed!, + Maxwell

$$\Rightarrow \boxed{\begin{aligned} \omega^2 &= \omega_p^2 \\ \omega^2 &= \omega_p^2 + c^2 k^2 \end{aligned}}$$

plasma waves

em waves

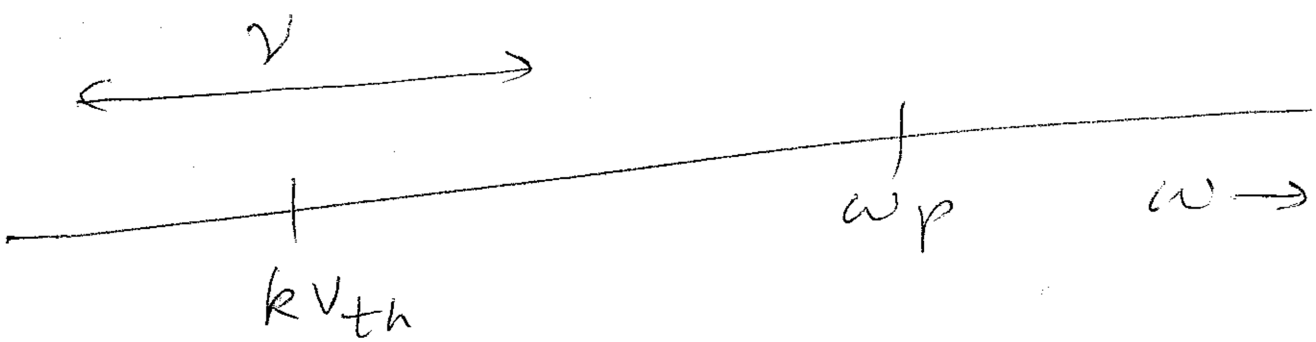
self consistent with the ansatz

$k v_{th} << \omega$ provided $k\lambda_D << 1$.

For Landau damping, $kv = \omega$ $\Rightarrow$ small population $\rightarrow$ should be reconsidered

Low frequency Plasma Response

$(k\lambda_D \ll 1)$

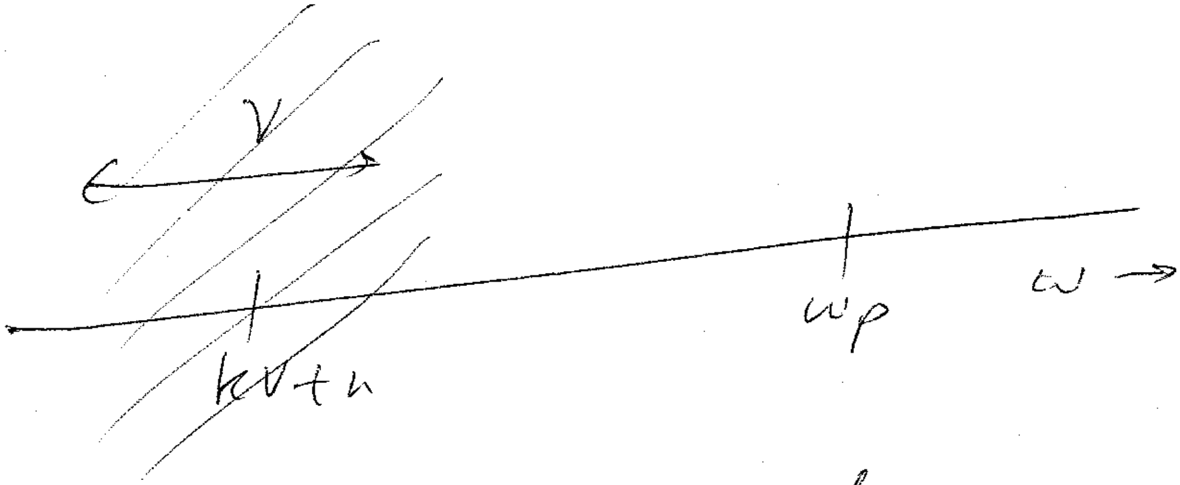


For simplicity, restrict to electrostatic response only. Thus, equation system is

$$\boxed{\begin{array}{ll} (1) & \dfrac{\partial f_\alpha}{\partial t} + \vec{v}\cdot\vec{\nabla} f_\alpha \mp \left(\dfrac{e}{m}\right)_\alpha \vec{\nabla}\varphi\cdot\dfrac{\partial f_\alpha}{\partial \vec{v}} = C(f) \\ & \{f_\alpha, \varphi\} \text{ system} \\ (2) & \text{close with } \nabla^2\varphi = -\sum_\alpha \int d\vec{v}\, e_\alpha f_\alpha \end{array}}$$

Fundamental eqns

Quasineutrality – the first step



$k\lambda_D \ll 1$ assumed

$\omega \ll \omega_p$, $\omega \sim kv_{th} \sim \nu$ for now

Consider this ordering

we want inertia and charge to feature together so we might assume in $\mathcal{O}$ that

$$\frac{\partial f}{\partial t} \sim \frac{e}{m} \vec{\nabla}\phi \cdot \frac{\partial f}{\partial \vec{v}},$$

ie, these terms must be of the same order.

$\Rightarrow \omega \sim \frac{e}{m}\frac{\varphi}{L v_{th}}$, and $\omega \sim k v_{th}$

Now examine (2)

$$\nabla^2 \varphi = e(n_e - n_i^0)$$

$$\Rightarrow \frac{\varphi}{L^2} : en_e : en_i^0$$

Compare these, using

$$\Rightarrow \frac{\omega L v_{th} m}{eL^2} : en_e$$

$$\omega \sim k v_{th} \Rightarrow \frac{m v_{th}^2}{eL^2} : en_e \Rightarrow \frac{T}{n_e e^2} : L^2$$

$$\Rightarrow \lambda_D^2 : L^2 \leftarrow \text{comparison}$$

small!

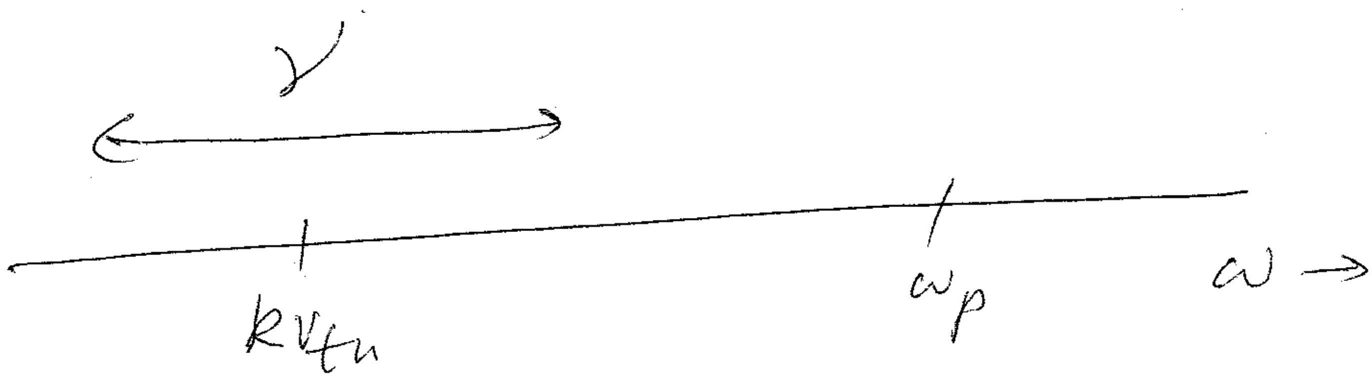
$$\Rightarrow |\nabla^2 \varphi| \ll |en_e| \Rightarrow 0 \simeq e(n_e - n_i^0)$$

(SPRING is not ... suppressed!) $\Rightarrow \boxed{n_e \simeq n_i^0}$, quasineutrality

$\therefore$ "Reduced" system for $\omega \ll \omega_p$ response

$$\boxed{\begin{gathered}\frac{\partial f_\alpha}{\partial t} + \vec{v}\cdot\vec{\nabla} f_\alpha - \left(\frac{e}{m}\right)_\alpha \vec{\nabla}\phi \cdot \frac{\partial f_\alpha}{\partial \vec{v}} = C(f) \\ \sum_\alpha \int d\vec{v}\, e f_\alpha \simeq 0\end{gathered}}$$

Now consider $\nu : \omega \sim k v_{th}$



Case ① $\nu \gg k v_{th}$ $\iff$ $\lambda \ll L$, coll'nl

Case ② $\nu \ll k v_{th}$ $\iff$ $L \ll \lambda$, coll'less

Collisional Low Frequency Response

Use Chapman–Enskog!

$$\Rightarrow \quad \frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha u_\alpha) = 0 \quad (3)$$

isotropic

$$\frac{\partial (nmu)_\alpha}{\partial t} + \frac{\partial}{\partial x}(nmuu)_\alpha = -\frac{\partial p_\alpha}{\partial x} - (ne)_\alpha \frac{\partial \phi}{\partial x} \quad (4)$$

$$\frac{\partial p_\alpha}{\partial t} + \frac{u_\alpha \partial p_\alpha}{\partial x} + \gamma p_\alpha \frac{\partial u_\alpha}{\partial x} = 0 \quad (5)$$

$$n_e \simeq n_i \quad \text{or} \quad \sum_\alpha e_\alpha n_\alpha = 0$$

$\{n_\alpha, u_\alpha, p_\alpha, \phi\}$ system, <u>complete</u>.

<u>Further simplification</u> use $n_e \simeq n_i$

Sum $\sum_\alpha e_\alpha (3) \Rightarrow \frac{\partial}{\partial x} n e(u_e - u_i) \simeq 0$

$$\Rightarrow \boxed{\begin{array}{l} u_e \simeq u_i = u \\ n_e \simeq n_i = n \end{array}}$$

Sum $\sum_\alpha (4) \Rightarrow \frac{\partial}{\partial t}(nu\Sigma m_\alpha) + \frac{\partial}{\partial x}(nuu\sum_\alpha m_\alpha)$

$$= -\frac{\partial}{\partial x}\Sigma p_\alpha$$

let $\sum_\alpha M_\alpha \longrightarrow M$ $\qquad p \equiv \sum_\alpha p_\alpha$

let $m_e \longrightarrow 0$

$$\Longrightarrow \quad \frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0 \qquad (6)$$

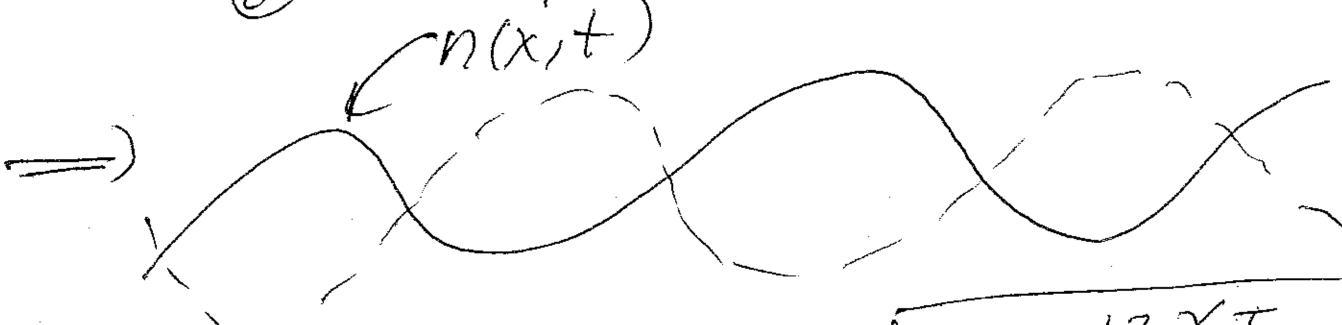
$$nM \frac{du}{dt} = -\frac{\partial p}{\partial x} \qquad (7)$$

$$0 = -\frac{\partial p_e}{\partial x} - ne\frac{\partial \varphi}{\partial x} \qquad (8)$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \gamma p \frac{\partial u}{\partial x} = 0 \qquad (9)$$

$\{(6), (7), (9)\}$ complete for $\{n, u, p\}$

$(8) \Longrightarrow \varphi$ ("adiabatic" electrons)



sound waves, $\boxed{\omega^2 = k^2 \frac{\gamma T}{M}}$

Collisionless Low Frequency Response

$\Rightarrow$ must use Vlasov-Maxwell system with quasineutrality

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}\cdot\vec{\nabla} f_\alpha - \left(\frac{e}{m}\right)_\alpha \vec{\nabla}\varphi \cdot \frac{\partial f_\alpha}{\partial \vec{v}} = 0$$

$$\sum_\alpha \int d\vec{v}\, e_\alpha f_\alpha = 0$$

$$\Rightarrow \boxed{\omega^2 = k^2 v_{ti}^2\,)}$$ "ion-acoustic" waves

$\uparrow$ No γ

$\hookrightarrow T_e \gg T_i$; phase mixing/LD for $T_i \gtrsim T_e$