Can now solve for f_1 from ②

2nd order

$$C(f_2) = \frac{\partial f_1}{\partial t} + \vec{v} \cdot \vec{\nabla} f_1$$

Annhilators $\Rightarrow$ condition on f_1

Now, have moment eqns with $(f_0 + f_1)$ in them

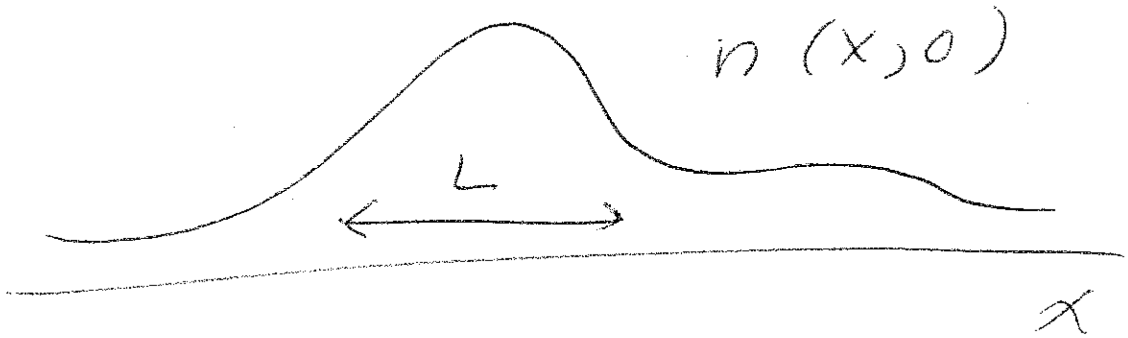
$\Rightarrow$ 1st order fluid eqns "(non-ideal)"

aka, NAVIER-STOKES eqns

$$\boxed{\begin{aligned} &\partial n/\partial t + \vec{\nabla} \cdot (n\vec{u}) = 0 \\ &nM\, d\vec{u}/dt = -\vec{\nabla} p + nM\mu\left[\tfrac{1}{3}\vec{\nabla}(\vec{\nabla}\cdot\vec{u}) + \vec{\nabla}^2\vec{u}\right] \\ &\frac{3}{2} n \frac{dT}{dt} + n\cancel{T}\,\vec{\nabla}\cdot\vec{u} = \vec{\nabla}\cdot(K\vec{\nabla}T) \end{aligned}}$$

μ = viscosity $\qquad K$ = thermal conductivity

Test problem for collisionless gas



Find $n(x,t)$

Suppose $\lambda \gg L$

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = C(f)$$

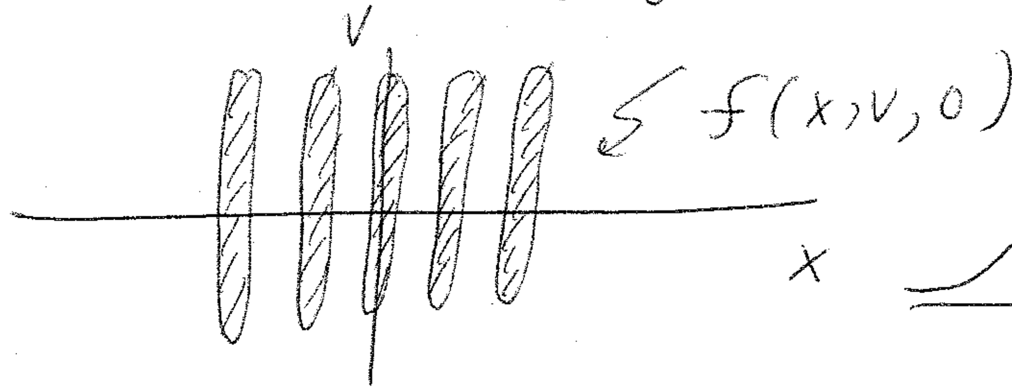
$$\omega : \frac{v_{th}}{L} : \nu$$

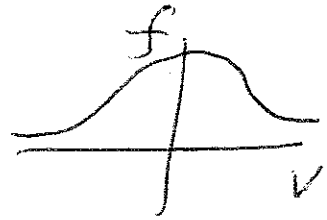
$$\Rightarrow \frac{\omega}{\nu} : \frac{\lambda}{L} : 1$$ small

$$\Rightarrow \boxed{\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = 0}$$

Let $n(x,0) = n_0 + \tilde{n}\cos(kx)$

ie, $f(x,v,0) = \dfrac{n_0}{(2\pi T_0)^{3/2}}\, e^{-\frac{m}{2}\frac{v^2}{T_0}}\left[1 + \dfrac{\tilde{n}}{n_0}\cos(kx)\right]$



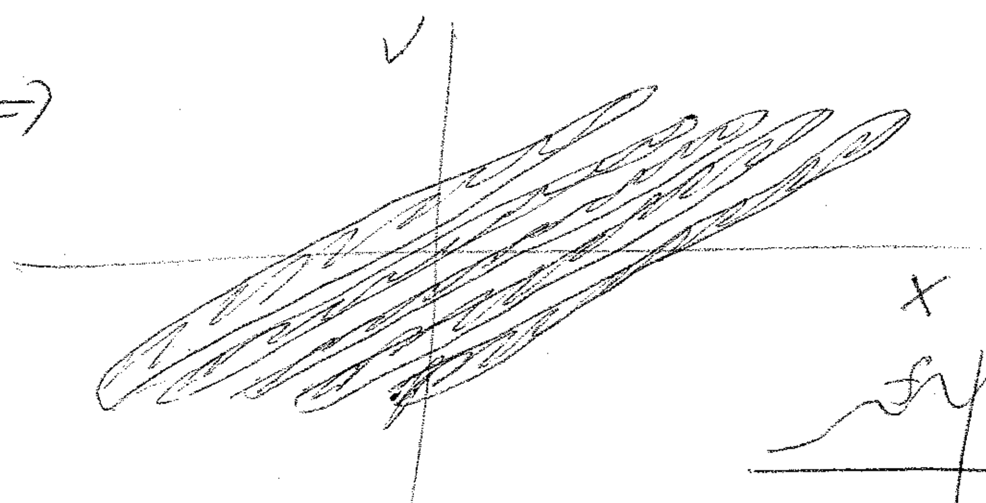


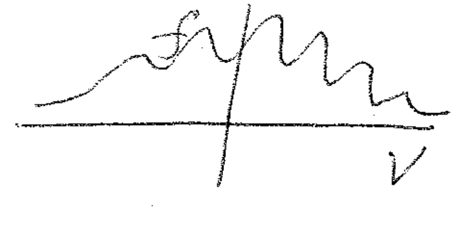
But $f(x-vt)$ is a solution

$\Rightarrow f(x,v,t) = f(x-vt, v, t)$

$$\Rightarrow f(x,v,t) = \frac{n_0}{(2\pi T_0)^{3/2}}\, e^{-\frac{m}{2}\frac{v^2}{T_0}}\left[1 + \frac{\tilde{n}}{n_0}\cos(k(x-vt))\right]$$

$\Rightarrow$





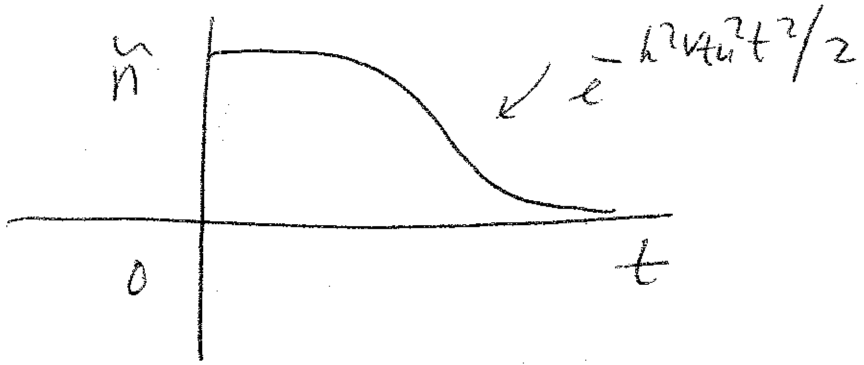
$$\Rightarrow \quad n(x,t) = n_0 + \frac{\tilde{n}}{(2\pi T_0)^{3/2}} \int_{-\infty}^{\infty} dv\, e^{-\frac{m}{2T}v^2} \cos[k(x-vt)]$$

$$2\cos[k(x-vt)] = e^{ik(x-vt)} + e^{-ik(x-vt)}$$

$$\int_{-\infty}^{\infty} dv\, e^{-\frac{m}{2T}v^2}\, e^{ikx}\, e^{-ikvt}$$

$$= e^{ikx} \int_{-\infty}^{\infty} dv\, e^{-\frac{m}{2T}\left(v + ikt\frac{T}{m}\right)^2} e^{-\frac{k^2t^2}{2}\frac{T}{m}}$$

$$\Rightarrow \boxed{n(x,t) - n_0 = \tilde{n}\, e^{-k^2 v_{th}^2 t^2/2} \cos(kx)}$$

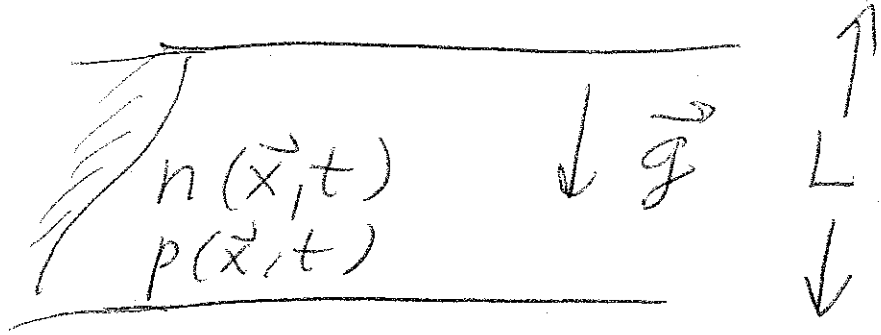


$\int dv\, f$ phase mixes out the striations

Reduced Equations

(An example)

Suppose



$$\Rightarrow \quad \frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{u}) = 0 \qquad (1)$$

$$nM\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\vec{\nabla}\vec{u}\right) = -\vec{\nabla}p + nM\vec{g} \qquad (2)$$

$$\frac{\partial p}{\partial t} + \vec{u}\cdot\vec{\nabla}p + \gamma p\,\vec{\nabla}\cdot\vec{u} = 0 \qquad (3)$$

$\{n, p, \vec{u}\}$ system

Suppose $p \gg nMgL$

$p \gg nMu^2$

[all energies are subsonic, Mach $\ll 1$]

Consider 2

$$nM\left(\frac{\partial \vec{u}}{\partial t}+\vec{u}\cdot\vec{\nabla}\vec{u}\right)=-\vec{\nabla}p+nM\vec{g}$$

$$\Rightarrow nM\omega uL : nMu^2 : p : nMgL$$

$$\Rightarrow \epsilon : \epsilon : 1 : \epsilon$$

using $\omega \sim u/L$

Expand in $p \to \infty$

$$2 \Rightarrow 0 = -\vec{\nabla}p_0$$

$$\Rightarrow p_0 = \text{const}$$

use this in 3 $\Rightarrow$

$$\frac{\partial p_0}{\partial t}+\vec{u}\cdot\vec{\nabla}p_0+\gamma p_0\vec{\nabla}\cdot\vec{u}=0$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{u}\simeq 0} \quad 4$$

use this in 1 $\Rightarrow$

$$\boxed{\frac{\partial n}{\partial t} + \vec{u}\cdot\vec{\nabla} n = 0} \quad (5)$$

Write 2 to first order $\Rightarrow$

$$nM\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\vec{\nabla}\vec{u}\right) = -\vec{\nabla}p_1 + nM\vec{g}$$

<u>Annihilate</u> with $\vec{\nabla}\times\{\ \}$

$$\Rightarrow \boxed{\vec{\nabla}\times nM\frac{d\vec{u}}{dt} = \vec{\nabla}(nM)\times\vec{g}} \quad (6)$$

$\{4, 5, 6\}$ is a <u>complete</u> system for $\{n, \vec{u}\}$ 4 unknowns

$\rightarrow$ 4 eqns since $\vec{\nabla}\cdot(6) = 0$

$\boxed{\text{REDUCED EQNS}} \Leftrightarrow \omega \ll kc_s$

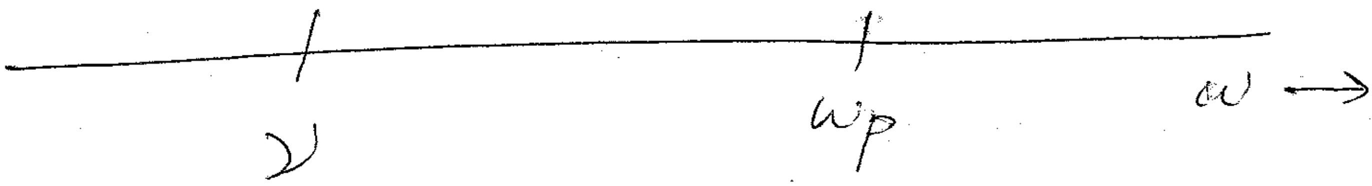
"Boussinesq appx"!

Test problem for plasmas

- Braginskii' eqns
- cold Fluid eqns
- Vlasov-Maxwell system
- Quasineutrality
-

Plasma – The Frequency Scale

$$\boxed{\nu \ll \omega_p \text{ , by definition}}$$



Fundamental equations

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}\cdot\vec{\nabla} f_\alpha + \frac{e_\alpha}{m_\alpha}\left(\vec{E} + \vec{v}\times\vec{B}\right)\cdot\frac{\partial f_\alpha}{\partial \vec{v}} = \sum C_{\alpha\beta}(f_\alpha, f_\beta)$$

$$\Rightarrow \{ f_\alpha, \vec{E}, \vec{B} \}.$$

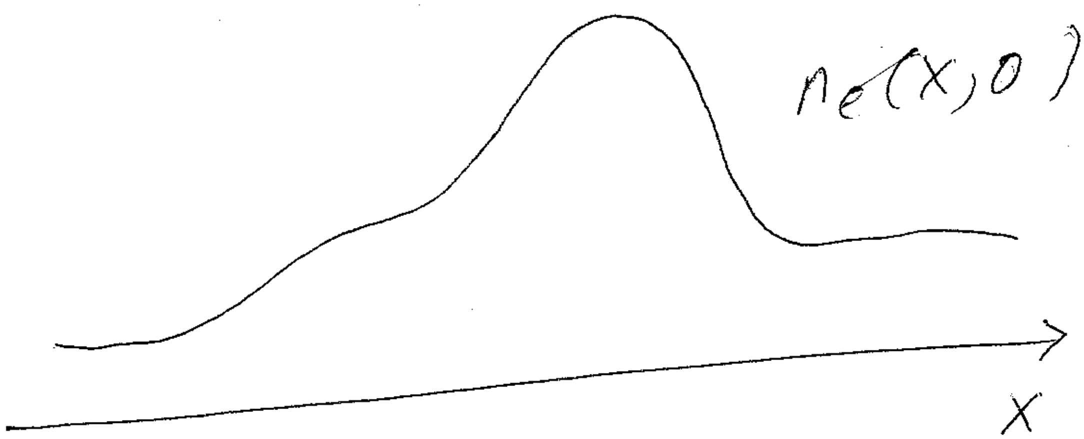
Close with Maxwell

$$\partial \vec{B}/\partial t = -\vec{\nabla}\times\vec{E}, \quad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \frac{\partial \vec{E}}{\partial t} + \sum_\alpha e_\alpha \int d\vec{v}\, \vec{v} f_\alpha$$

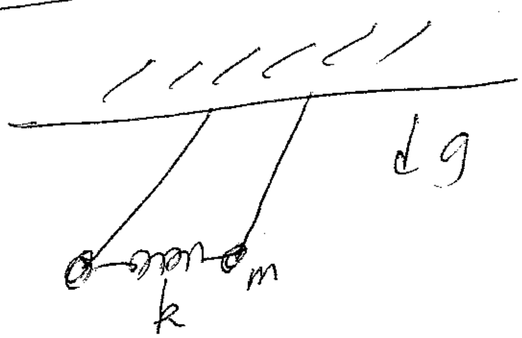
$$\vec{\nabla}\cdot\vec{E} = \sum_\alpha \int d\vec{v}\, e_\alpha f_\alpha$$

Response to test problem in an unmagnetized plasma

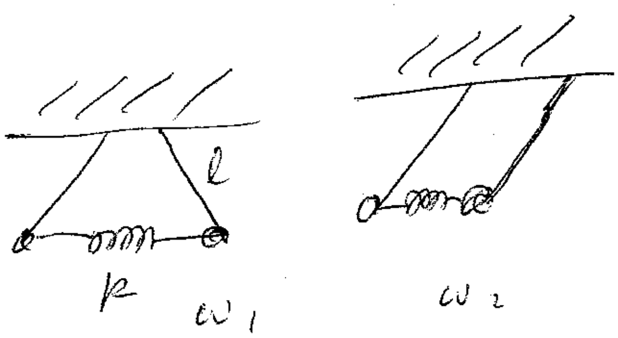


What about $n_i'(x,0)$?

Answer: Situation is like coupled pendula



Suppose $k \to \infty$



$\Rightarrow \omega_1^2 \simeq k/m$; compressible

$\omega_2^2 \simeq g/l$; incompressible

DISPARATE