Some Applications of $B_z \to \infty$ Reduced Eqns

(Must have $B_z = B_0$)

Equil $\quad B_z = B_0, \; \vec{B}_\perp = 0, \; n = n_0$

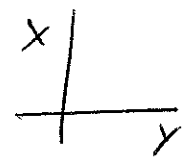
$$\varphi = 0, \; \psi = 0$$

Perturb $\quad \omega \hat{n} = 0$

$$\hat{z} \cdot \vec{\nabla}_\perp \times (\partial_t \hat{\vec{u}}_\perp) = B_0 \partial_z \nabla_\perp^2 \hat{\psi}$$

$$\omega \hat{\psi} = B_0 \partial_z \hat{\varphi}$$

$$\omega \nabla_\perp^2 \hat{\varphi}$$

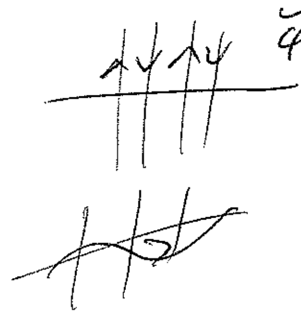


$$\partial_z \to i k_z, \quad \vec{\nabla}_\perp \to i k_y$$

$$k_y^2 (\hat{\varphi} \omega = B_0 k_z \hat{\psi})$$

$$\omega \hat{\psi} = B_0 k_z \hat{\varphi} \quad \Rightarrow \quad \boxed{\omega^2 = k_z^2 v_A^2}$$

$$\hat{\varphi}, \hat{\psi} \neq 0$$



Supports Alfven waves

but subMagsonic

Add $\vec{g}$

equil $\hat{z}\cdot\vec{\nabla}_\perp n\times\vec{g}=0 \Rightarrow n(x)$ only

$\varphi=0,\ \Psi=0$ $\qquad n(x)$

Perturb

$\omega\tilde{n}=n'\tilde{\varphi}k$

$\cancel{\omega\nabla_\perp^2\tilde{\varphi}=}$

$\omega\nabla_\perp\cdot(n\nabla_\perp\tilde{\varphi})=gk\tilde{n}+B_0\partial_z\nabla^2\tilde{\Psi}$

$\omega\tilde{\Psi}=k_z\tilde{\varphi}$

$e^{ik_yy}e^{ik_zz}$

$k_z=0$ $\Rightarrow\tilde{\Psi}=0 \Rightarrow \omega\nabla_\perp\cdot(n\nabla_\perp\tilde{\varphi})=gk\tilde{n}$

$$\boxed{\vec{\nabla}_\perp\cdot(n\vec{\nabla}_\perp\tilde{\varphi})=\frac{gk^2}{\omega^2}n'\tilde{\varphi}}$$

$\Rightarrow \boxed{\omega^2=gn'/n}$ same as fluids!

$k_z\neq0$ $\qquad \omega^2\tilde{\varphi}=g\cancel{k}n'\tilde{\varphi}+k_z\tilde{\Psi}\omega$

$\omega\tilde{\Psi}=k_z\tilde{\Psi}$

$$\Rightarrow \boxed{(\omega^2+gn'/n)=k_z^2v_A^2}$$

stabilizing

$\therefore$ selects $k_z=0$

B10

Linetying & B.C.

Conducting wall $\Rightarrow \tilde{\vec{E}}_\perp = 0$

$$\vec{E} = -\vec{u} \times \vec{B}$$

$$\hat{z} \times \vec{E} = B_z \vec{u} - u_z \vec{B}$$

$$\Rightarrow B_0 \vec{u}_\perp = u_z \vec{B}_\perp \Rightarrow B_0 \tilde{\vec{u}}_\perp = 0$$

$$\Rightarrow \hat{z} \times \vec{\nabla}_\perp \tilde{\varphi} = 0 \Rightarrow k_y \tilde{\varphi} = 0 \Rightarrow \boxed{\tilde{\varphi} = 0}$$

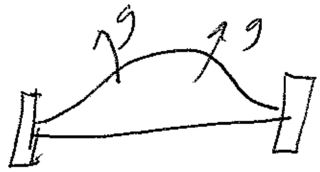
$$\omega^2 \nabla_\perp^2 \tilde{\varphi} = g n' \tilde{\varphi} + \partial_z^2 \tilde{\varphi}$$

$$\tilde{\varphi} \to \sin(k_n z)$$

$$\Rightarrow (\omega^2 + g n'/n) = k_{zn}^2 \qquad n = 1, \ldots$$

$\Rightarrow$ stabilization by linetying

Original MCX



Solar loops