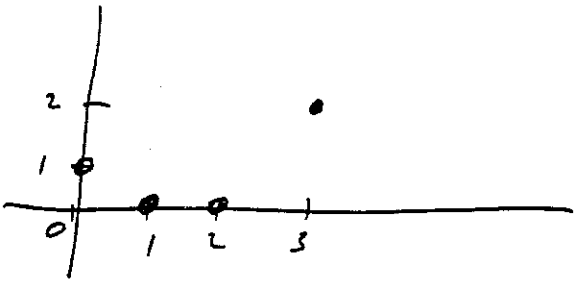
Phys 761/F96/Hanany/Soln to Problem 3a

$$\epsilon^2 x^3 - x^2 + x - \epsilon = 0, \qquad \epsilon \ll 1$$

Newton's Method



$\Rightarrow$ $2 \rightarrow 3$

$1 \rightarrow 2$

$1 \rightarrow 1$

(a) Lowest order

$$\boxed{X_{10} = \epsilon}$$

$\Leftrightarrow$ $\epsilon^2 x^3, x^2 \ll \epsilon$

$\Rightarrow$ $\epsilon^5, \epsilon^2 \ll \epsilon \Rightarrow \epsilon \ll 1$ OK

$$\boxed{X_{20} = 1}$$

$\Leftrightarrow$ $\epsilon^2 x^3, \epsilon \ll x$

$\Rightarrow$ $\epsilon^2, \epsilon \ll 1$ OK

$$\boxed{X_{30} = 1/\epsilon^2}$$

$\Leftrightarrow$ $x, \epsilon \ll x^2 \Rightarrow \frac{1}{\epsilon^2}, \epsilon \ll \frac{1}{\epsilon^4} \Leftrightarrow \epsilon^2, \epsilon^5 \ll 1$ OK

(b) Next order Do iteratively (although doing it by $x_0 + x_1 + x_2 \ldots$ also ok)

Iteration on x_{10}

$$x - \epsilon = -\epsilon^2 x^3 + x^2, \quad x_{10} \backsimeq \epsilon$$

$$\Rightarrow x_1 - \epsilon \backsimeq -\epsilon^2 \epsilon^3 + \epsilon^2$$

$$\Rightarrow \boxed{x_1 \backsimeq \epsilon + \epsilon^2}$$

Iterate on x_{20}

$$-x^2 + x = \epsilon - \epsilon^2 x^3$$

$$\Rightarrow x - 1 \backsimeq -\frac{\epsilon}{x} + \epsilon^2 x^2, \quad x_{20} = 1$$

$$\Rightarrow x_2 - 1 \backsimeq -\epsilon + \epsilon^2 \Rightarrow \boxed{x_2 \backsimeq 1 - \epsilon}$$

Iterate on x_{30}

$$\epsilon^2 x^3 - x^2 = -x + \epsilon$$

$$\Rightarrow x - \frac{1}{\epsilon^2} = -\frac{1}{x\epsilon^2} + \frac{1}{x^2 \epsilon}, \quad x_{30} = 1/\epsilon^2$$

$$\Rightarrow x_3 - \frac{1}{\epsilon^2} \backsimeq -1 + \epsilon^3$$

$$\Rightarrow \boxed{x_3 \backsimeq \frac{1}{\epsilon^2} - 1}$$