Problem #4 Reduced eqns, 1D, on transport timescales

$u = u_x$

① $\partial_t n + \partial_x (nu) = 0$

② $nM d_t u = - \partial_x p$ + viscous terms

③ $\partial_t p + u \partial_x p + \gamma p \partial_x u = \partial_x (\kappa \partial_x T)$

$$p \equiv nT \, , \quad \kappa \equiv n \chi(n,T)$$

- $\{u_y, u_z\}$ have separate eqns but do not ~~couple~~ influence $\{n, u, p\}$ system.
- Above system supports sound waves as well as viscous effects + thermal conduction.
- Scaling let $\partial_t \ll c_s / L$, $\partial_t \sim u/L$

  Note $\frac{\kappa}{n} \sim \mu \sim \lambda c_s$, $\lambda / L \ll 1$

①-③ $\Rightarrow$

$$\begin{matrix} & 1 & : & 1 & & \\ \partial_t u & : & \frac{u^2}{L} & : & \frac{c_s^2}{L} & : & \frac{\lambda c_s u}{L^2} \end{matrix} \Rightarrow \epsilon^2 : \epsilon^2 : 1 : \epsilon^2$$

$$1 : 1 : 1 : 1$$

where $\epsilon \equiv \frac{u}{c_s} \sim \frac{\lambda}{L}$. We take $\partial_t \sim \frac{\lambda}{L} \frac{c_s}{L}$ as "optimal ordering"

<u>lowest order</u>: (drop all subscripts, ie, in $p = p_0 + p_1 + \cdots$, $p_0 \to p$, etc)

$$\Rightarrow \partial_t n + \partial_x(nu) = 0 \quad (1')$$

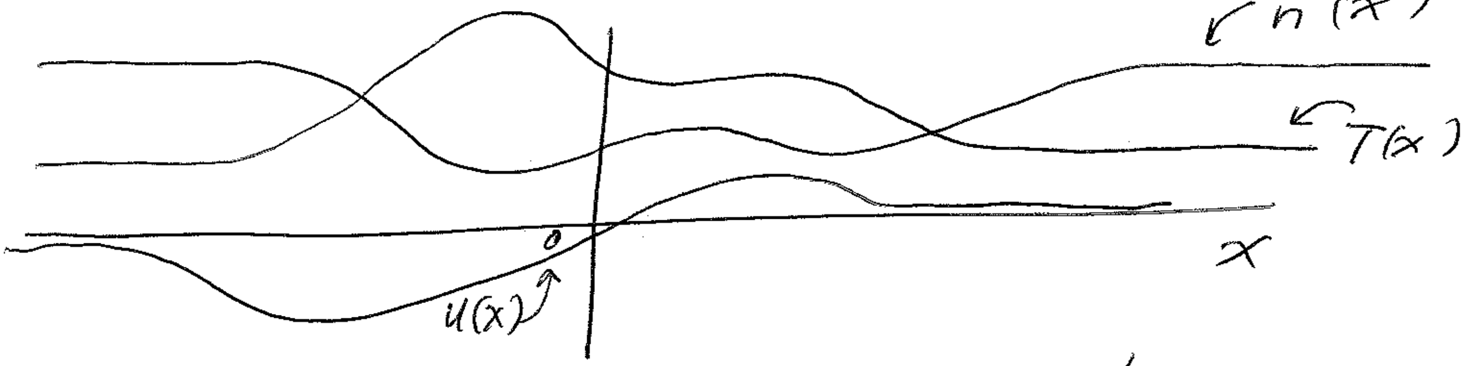
$$0 = -\partial_x p \quad (2')$$

$$\partial_t p + u\partial_x p + \gamma p \partial_x u = \partial_x(\kappa \partial_x T) \quad (3')$$

(2') $\Rightarrow$ $p(x,t) = p_0 = \text{const.}$
[assume cannot be $p_0(t)$]

(3') $\Rightarrow$ $\gamma p_0 \partial_x u = \partial_x(\kappa \partial_x T)$

let us assume profiles of type



ie, assume $n(|x| \to \infty) \to \text{const}$
$T(|x| \to \infty) \to \text{const}$
$u(|x| \to \infty) \to 0$

Then $\gamma p_0 u = \kappa \partial_x T + \text{const}$

but const $\to 0$ from b.c. at ∞

$$\Rightarrow \boxed{u = \frac{\kappa}{\gamma p_0}(\partial_x T)}$$

Then, $\partial_t n + u\partial_x n + n\partial_x u = 0$ ④

$nT \equiv p_0$, $u = \frac{\kappa}{\gamma p_0}\partial_x T$ ⑤

To find eqn for T, eliminate n in ④

④ $\Rightarrow$ $\partial_t T + u\partial_x T = T\partial_x u$

eqn for $T(x,t)$. NL eqn.

$$\Rightarrow \boxed{\begin{gathered} \partial_t T + u\partial_x T = \frac{T}{\gamma}\partial_x\left[\frac{\chi}{T}\partial_x T\right] \\ u \equiv \frac{\chi}{\gamma T}(\partial_x T) \end{gathered}}$$

where $K \equiv n\chi(n,T) \rightarrow \frac{p_0}{T}\chi\left[\frac{p_0}{T}, T\right]$

<u>Linearize</u> suppose $T = T_0 + \tilde{T}$, $\tilde{T} \ll T_0$
$T_0 = \text{const.}$, $n_0 = p_0/T_0$

$$\Rightarrow \boxed{\partial_t \tilde{T} \simeq \frac{\chi_0}{\gamma}\partial_x^2 \tilde{T}}$$

Thus, heat conduction coefficient $\sim \frac{\chi_0}{\gamma}$

$\xrightarrow{\hspace{2em}}$ unexpected factor