Phys 767 / S07 / Hassam

Problem (2)

The ideal fluid equations are

$$\partial n/\partial t + \vec{\nabla}\cdot(n\vec{u}) = 0 \qquad (1)$$

$$nM\, d\vec{u}/dt = -\vec{\nabla}p \qquad (2)$$

$$dp/dt + \gamma p\, \vec{\nabla}\cdot\vec{u} = 0 \qquad (3)$$

$$d/dt \equiv (\partial/\partial t + \vec{u}\cdot\vec{\nabla}), \quad \gamma \equiv 5/3.$$

Using (1) + (2), show that (3) can be rewritten in the two equivalent forms

$$\frac{d}{dt}\left(\frac{p}{n^{\gamma}}\right) = 0 \qquad (3')$$

$$\frac{\partial}{\partial t}\mathcal{E} + \vec{\nabla}\cdot(\mathcal{E}\vec{u}) = -\vec{\nabla}\cdot(p\vec{u}) \qquad (3'')$$

$$\mathcal{E} \equiv \frac{1}{2}nMu^2 + \frac{3}{2}p$$