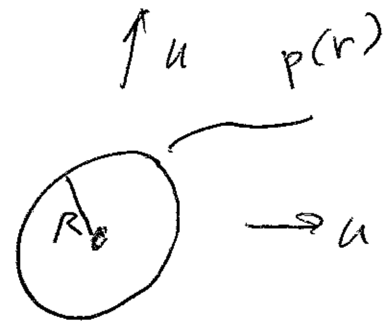
Soln to #11 — Parker's Solar Wind



Equilibrium

$$n M_p \vec{u}\cdot\vec{\nabla}\vec{u} + \vec{\nabla} p = -\hat{r}\,\frac{G M n M_p}{r^2}$$

① $\vec{u} \to u\hat{r} \Rightarrow n M_p u u' + n' T = -\frac{G M n M_p}{r^2}$

$T = T_0 = \text{const}$

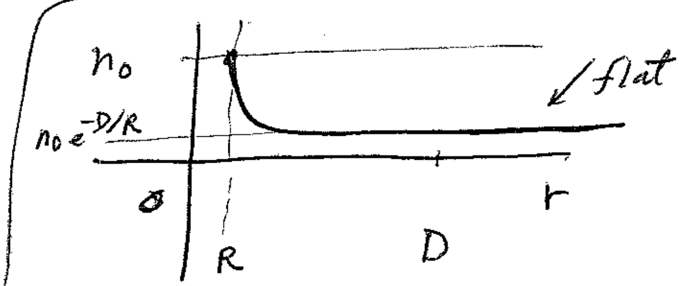
$$\Rightarrow \boxed{u u' + \frac{n'}{n}\frac{T}{M_p} = -\frac{GM}{r^2}} \quad ①$$

Also, $\vec{\nabla}\cdot(n\vec{u}) = 0 \Rightarrow \frac{1}{r^2}\frac{d}{dr}(r^2 n u) = 0$

$$\Rightarrow \boxed{n u r^2 = F = \text{const}} \quad ②$$

② $\underline{u = 0} \Rightarrow \frac{n'}{n} = -\frac{D}{r^2}$ $\qquad \boxed{D \equiv \frac{GM}{(T/M_p)}}$

$$\Rightarrow \ln n = \frac{D}{r} + \text{Const} \Rightarrow \boxed{n = n_0 \exp\left[\frac{D}{r} - \frac{D}{R}\right]}$$



Note; drops off quickly to $n_0 e^{-D/R}$ for $r \gg R$.

Note $D \gg R$

$$D = \frac{GM}{(T/M_p)} = \frac{GM}{c^2}\,\frac{c^2}{(T/M_p)}$$

$\sqrt{\frac{T}{M_p}} = 10^7$ cm/s @ $T = 100$ eV

$\frac{c}{\sqrt{T/M_p}} = 3\,10^3$, $\frac{2GM}{c^2}$ = Schwarschild Radius $= 3$ km

$R = 7\,10^5$ km

$\frac{D}{R} \simeq 20$, $\exp(-D/R) \simeq \frac{2}{10^9}$, $\boxed{n_\infty \to 20}$ ↖ large!

③ $\underline{u \neq 0}$ use ② → ① ⟹

$$uu' + \frac{T}{Mp}(ur^2)\left(\frac{1}{ur^2}\right)' = -\frac{GM}{r^2}$$

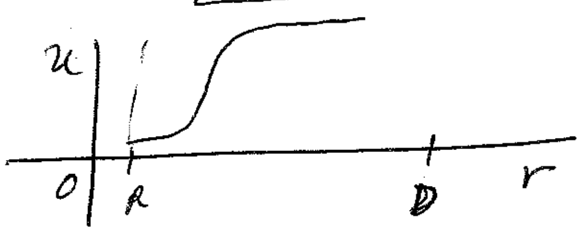
$\boxed{T/Mp = 1,\ D = 1}$ ⟹ $uu' + (ur^2)\left(\frac{1}{ur^2}\right)' = -\frac{1}{r^2}$

⟹ $uu' - \frac{u'}{u} - \frac{2}{r} = -\frac{1}{r^2}$ ⟹ $\boxed{u'\left(u - \frac{1}{u}\right) = \frac{2}{r} - \frac{1}{r^2}}$ $u(r)$

④ $\underline{u \ll 1,\ r \ll 1}$ $\underline{r \gtrsim R \ll 1}$

⟹ $+\frac{u'}{u} \simeq +\frac{1}{r^2}$ ⟹ $\ln u = -\frac{1}{r} + C$ ⟹ $u = Ce^{-1/r}$

$u(R) \to u_0 \ll 1$ ⟹ $\boxed{u \simeq u_0 e^{\left(\frac{1}{R} - \frac{1}{r}\right)}}$ $R \ll 1$

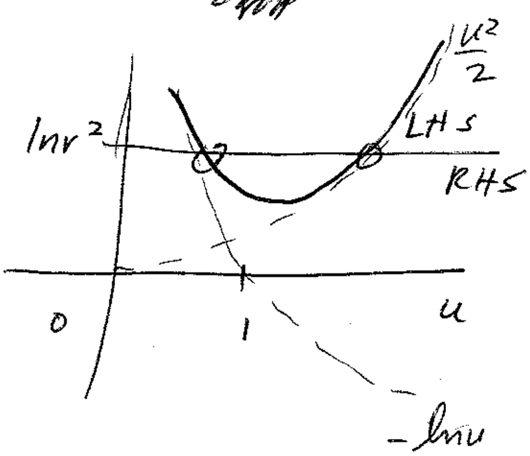


⑤ $\underline{1 \ll r}$ $u \sim 1$

⟹ $u'\left(u - \frac{1}{u}\right) = \frac{2}{r}$ ⟹ $\frac{u^2}{2} - \ln u = +2\ln r + C$

$r \to \infty$ ⟹ $\frac{u^2}{2} - \ln u = +\ln r^2$

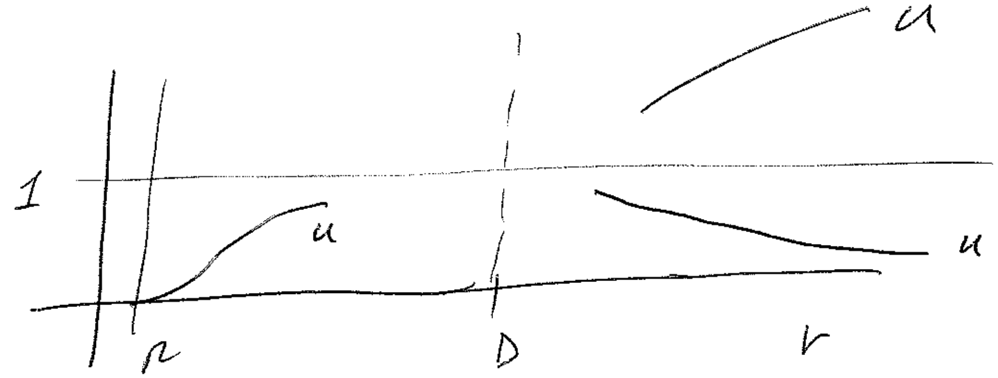
$r \to \infty$ ⟹ $\frac{u^2}{2} \simeq \ln r^2$, or $u \to \frac{c}{r^2}$



$r^2 n u = \text{const}$ ⟹ $n \to \frac{1}{\sqrt{\ln r^4}\, r^2}$

or $n \to \text{const}$

(6)



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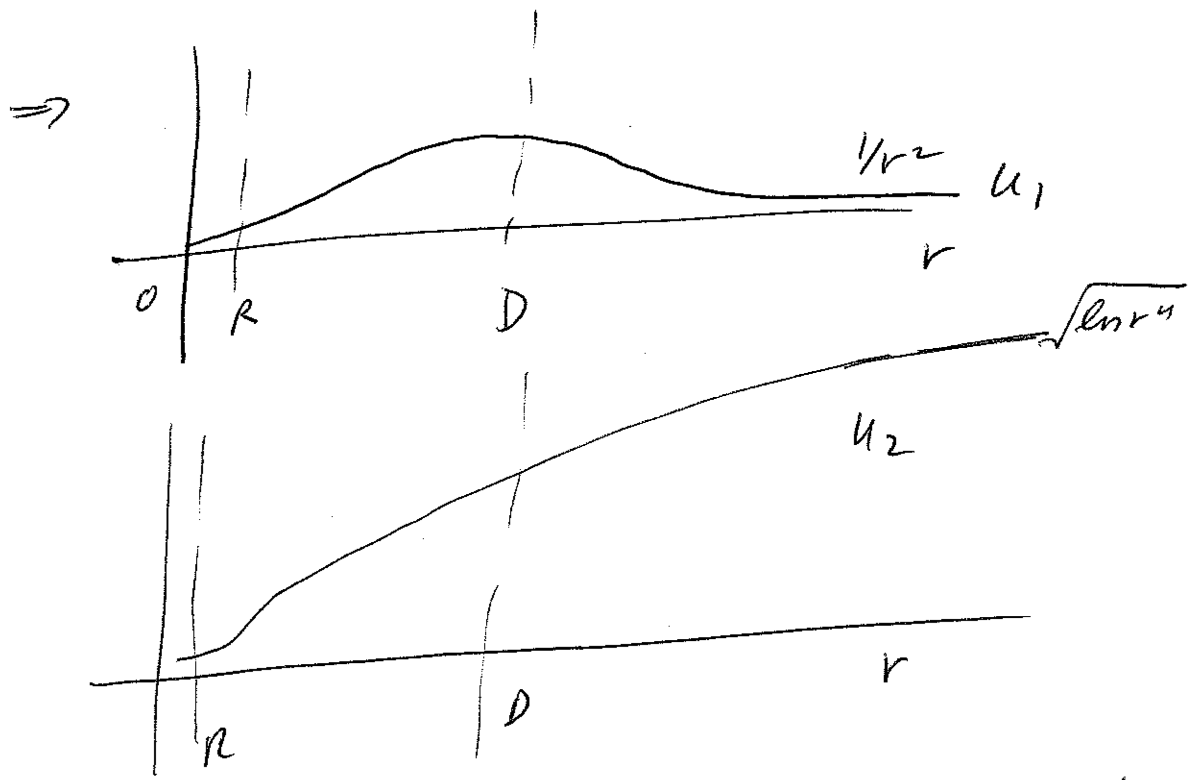

(7) u_1 is not viable since $n \to$ const, ~~same as~~ similar to $u \to 0$ soln

@ 1 AU ≅ 200 R, u_2 is sonic or supersonic

$$\frac{u_{1AU}}{\sqrt{T/M_p}} \sim \frac{400\ \text{km/s}}{10^2\ \text{km/s}} \sim \text{Mach } 4$$

Observed is ~ Mach 10