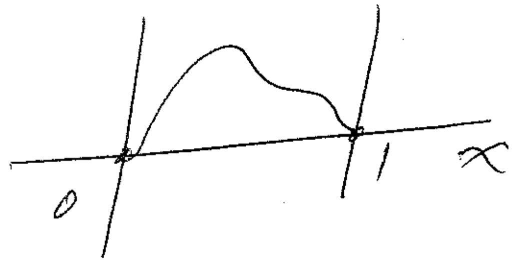
Soln to P10

$$\epsilon y'' + 2y' + e^{y} = 0, \quad y(0)=0, \quad y(1)=0$$

Naive $\xrightarrow{\text{from here}}$



suppose $\frac{d}{dx} \sim \frac{1}{1}$

$$\Rightarrow \quad \epsilon : 1 : \frac{e^{y}}{y}$$

3d term is $\gtrsim O(1) \quad \forall y$

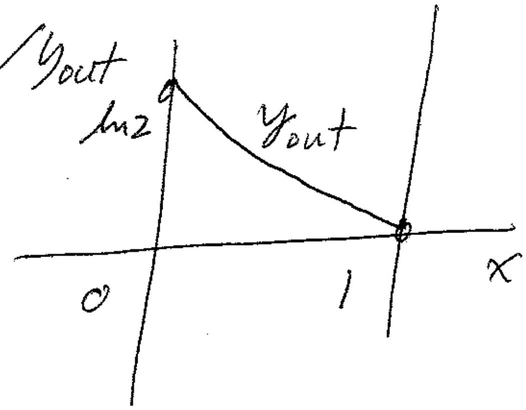
2nd term is always larger than 1st

$$\therefore \quad 2y' + e^{y} \simeq 0$$

$$\Rightarrow \quad dy\, e^{-y} = -\frac{1}{2} dx \Rightarrow e^{-y} = \frac{1}{2}x + C$$

$$\Rightarrow \quad 1 = \frac{1}{2} + C \quad \Rightarrow \quad C = 1/2$$

$$\Rightarrow \quad \boxed{y_{out}(x) = \ln\left(\frac{2}{x+1}\right)}$$



But $y(0) \neq 0$

$y(0) = \ln 2$

$\therefore$ BL as $x \to 0$

Suppose $\frac{\cancel{d}d}{dx} \sim \frac{1}{x}$, $x \ll 1$

$$\Rightarrow \frac{\epsilon}{x^2} : \frac{2}{x} : \frac{e^y}{y}$$

From y_{out}, $y \lesssim O(1)$.
if $y \sim O(1)$, 3d term smaller than 2nd
if $y \ll O(1)$, 3d term could dominate second, depending. Take a chance and <u>ansatz</u> that $\boxed{|2y'| \gg e^y}$ check later

$$\Rightarrow \epsilon y'' + 2y' \simeq 0$$

$$\Rightarrow \epsilon y' + 2y = C \quad \Rightarrow y = C$$

or $y = De^{-2x/\epsilon}$

$$y(0) = 0 \Rightarrow \boxed{y_{in}(x) = C\left(1 - e^{-2x/\epsilon}\right)}$$

Asymptotic Matching:

$$y_{in}(\epsilon \ll x \cancel{\ll} 1) \longleftrightarrow y_{out}(\epsilon \ll x \ll 1)$$

$\Leftrightarrow \quad C \leftrightarrow \ln 2$

$$\Rightarrow \boxed{\begin{array}{lll} y_{in}(x) = \ln 2 \left(1 - e^{-2x/\epsilon}\right) & & \epsilon \sim x \ll 1 \\ y_{out}(x) = \ln\left(\frac{2}{x+1}\right) & & \epsilon \ll x \sim 1 \end{array}}$$

matched asymptotic expansion

Need to check the smallness of e^y.

<u>Demand</u> $\quad |2y'| \gg e^y$ for y_{in}

$$\Rightarrow \left|\frac{4}{\epsilon} \ln 2 \, e^{-2x/\epsilon}\right| \gg e^{y_{in}}$$

$$\Rightarrow \left|\ln\frac{1}{\epsilon} - \frac{2x}{\epsilon}\right| \gg \ln 2\left(1 - e^{-2x/\epsilon}\right)$$

for $\epsilon \sim x \ll 1$

if $x \gg \epsilon \Rightarrow \left|\frac{x}{\epsilon}\right| \gg 1$ OK, ie self-consistent

if $x \sim \epsilon \Rightarrow \ln\frac{1}{\epsilon} \gg O(1)$ OK see later

if $x \ll \epsilon \Rightarrow \ln\frac{1}{\epsilon} \gg \left(\frac{2x}{\epsilon}\right)\ln 2$ OK $*$

Thus, y_{in} is good for $\epsilon \sim x \ll 1$.

Also demand

$$\left|\epsilon y''_{out}\right| \ll \left|2y'_{out}\right|$$

~~As~~ $y = \ln\left(\frac{2}{1+x}\right)$, $y' = \frac{-1}{1+x}$, $y'' = \frac{1}{(1+x)^2}$

$$\Rightarrow \frac{\epsilon}{(1+x)^2} \ll \frac{2}{(1+x)}$$

$$\Rightarrow \epsilon \ll (1+x) \qquad \text{OK for } \epsilon \ll 1$$

[Seems like no further scale for y_{in}; I was misinterpreting*]