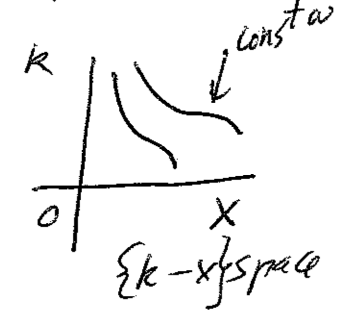
# ① 1-D Geometric Optics

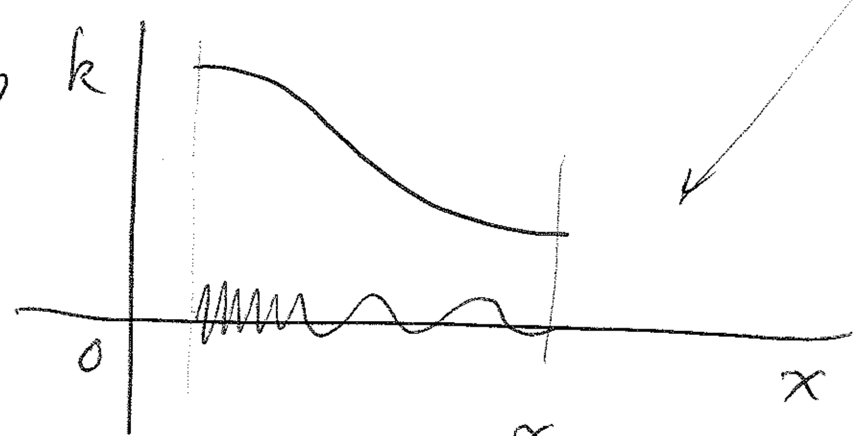
WDI




From 1-D, we learn

$k = k(\omega, x)$ ; fixed $\omega$

⇒




* we have here $k(x)$ or we can parameterize as $k(t), x(t)$; as below, let us Choose to parameterize such that $\dot{x} = v_g$; $x(t)$ ↑ definition

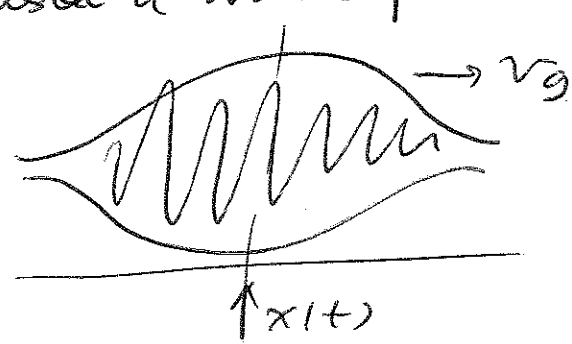
$$\Rightarrow e^{i\int^{x} k(x')\,dx' - i\omega t}$$

also, $\omega = \omega(k, x)$

fixed $\omega \Rightarrow k$ & $x$ vary

Now, we know that $v_g = \left(\frac{\partial\omega}{\partial k}\right)_x = v_g(x)$

Thus, let $x(t)$ label a wave packet*




$\Rightarrow \dot{x} \equiv v_g$

$\Rightarrow$ let $\boxed{\dot{x} \equiv \frac{\partial\omega}{\partial k}}$ ← $\frac{\partial\omega}{\partial k} = v_g[k(t), x(t)]$ ⇒ know $x(t)$

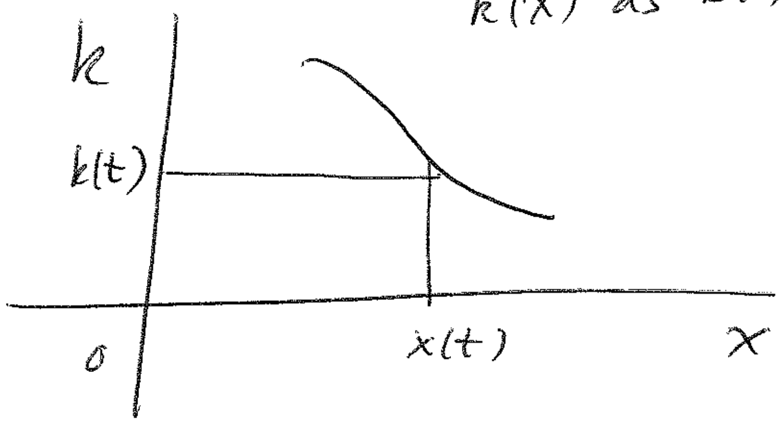
Then $\omega = \omega(k, x)$

$$\Rightarrow \quad \dot{\omega} = 0 = \frac{\partial \omega}{\partial k}\dot{k} + \frac{\partial \omega}{\partial x}\dot{x}$$

Note $k = k(t)$, ie, $k$ changes in $x$ which is $x(t)$

$$\Rightarrow 0 = \dot{x}\dot{k} + \frac{\partial \omega}{\partial x}\dot{x}$$

Aside: we are parameterizing $k(x)$ as $k(t), x(t)$

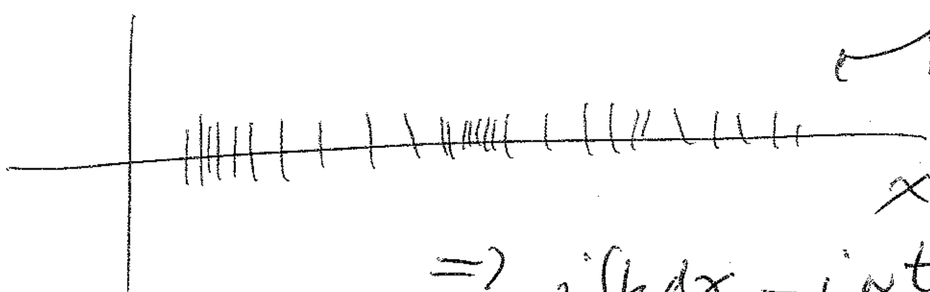



$\Rightarrow$ ($\dot{x} \neq 0$) $\boxed{\dot{k} = -\frac{\partial \omega}{\partial x}}$

AND $\boxed{\dot{x} = \frac{\partial \omega}{\partial k}}$ $\qquad \boxed{\omega = \omega(k, x) = \text{const}}$

by parameterization, we deduce $x(t)$ & motion which yields local $k(t) \Rightarrow$

wave fronts

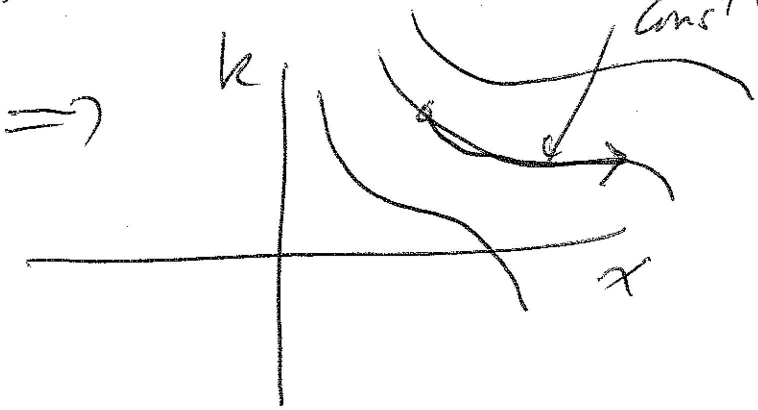



$$\Rightarrow e^{i\int k\,dx - i\omega t}$$

Note that $\{k, x\}$ satisfy

~~are~~ Hamiltonian eqns

$\Rightarrow$

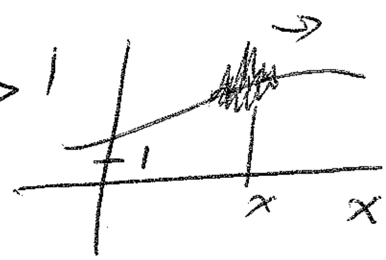



## Examples

Suppose propagn in dielectric

$$\epsilon(x)\partial_t^2\psi = \partial_x^2\psi \; ; \quad c = 1$$

$\epsilon > 1$




$\left|\frac{\epsilon'}{\epsilon}\right| \ll k$, sharp pulse

$$\Rightarrow \quad \epsilon\omega^2 = k^2$$

$$\boxed{\omega = \pm k/\sqrt{\epsilon}}$$

$$+ \quad \Rightarrow \quad \dot{x} = \frac{\partial\omega}{\partial k} = \frac{1}{\sqrt{\epsilon}}$$

$$\dot{k} = -\frac{\partial\omega}{\partial x} = \frac{\frac{1}{2}k}{\sqrt{\epsilon}}\,\frac{\epsilon'}{\epsilon}$$

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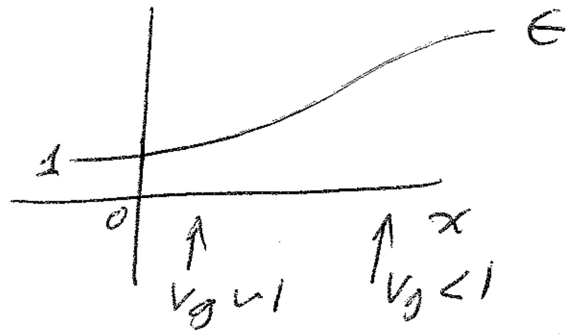
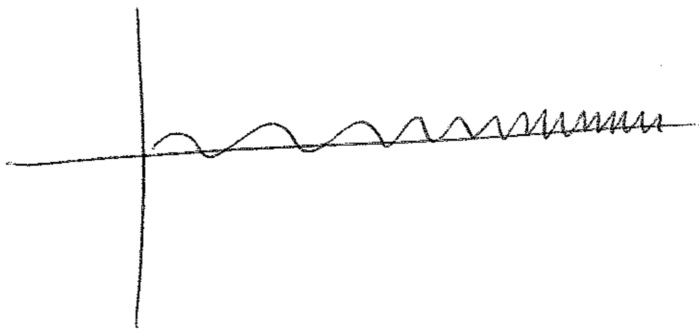
$x(t)$ decouples, $v_g = \frac{1}{\sqrt{\epsilon}} < 1$




$k = \sqrt{\epsilon}\,\omega$ from d.r.

$\therefore\ x \uparrow \Rightarrow \sqrt{\epsilon} \uparrow \Rightarrow k \uparrow$



Suppose Matter waves in $V(x)$

$$-i\hbar\partial_t\psi = \frac{\hbar^2\partial_x^2\psi}{2m} - V(x)\psi \quad , \psi(x,t)$$

$$e^{iS-i\omega t} \Rightarrow \boxed{\omega\hbar = +\frac{\hbar^2k^2}{2m} + V(x)}$$

$\omega(k,x)$

$$\hbar\partial\omega/\partial k = \frac{\hbar^2 k}{m} \; ; \; \hbar\partial\omega/\partial x = V'(x)$$

$$\Rightarrow \boxed{m\dot{x} = \hbar k \; ; \; \hbar\dot{k} = -V'(x)}$$ Newton's eqns of motion

Ehrenfest $\qquad kL \gg 1$

Note

$$\hbar\omega = \frac{\hbar^2 k^2}{2m} + V(x)$$

$$k = (dS/dx)$$

$$\Rightarrow \boxed{\hbar\omega = \frac{\hbar^2}{2m}\left(\frac{dS}{dx}\right)^2 + V(x)}$$

ODE for $S(x)$, 1st order

$\rightarrow$ This is H-J. eqn (1-D)