Phys374/F09/Hassam/PS 8 **due 10/27/09**

8.1

Consider the space of functions {f(x)} in the domain x=[0,1]. Any f(x) in this space has zero derivatives at the boundaries, i.e., f '(0) = 0 and f '(1) =0, where f ' = df/dx. Consider the operator $A = d^2/dx^2$ in this space.

(1) Show that A is a Hermitean operator.
(2) Find all the eigenfunctions, $f_n(x)$, and the eigenvalues, a_n, of this operator.
(3) Make sketches of the 4 eigenfunctions with the lowest eigenvalues.
(4) Consider the function g(x) defined as follows:

$$g(x) = +1, \qquad 0 < x < 1/2$$
$$g(x) = -1, \qquad 1/2 < x < 1$$

Write down the expansion of g(x) in the complete set $f_n(x)$ above. Find all the coefficients in the expansion.

(5) Consider the function g(x) defined as follows:

$$g(x) = +1, \qquad 0 < x < 1/2$$
$$g(x) = +1, \qquad 1/2 < x < 1$$

Write down the expansion of g(x) in the complete set $f_n(x)$ above. Find all the coefficients in the expansion.

8.2

Consider the vector space of functions f(x) in the domain x=[1,e] with f(1)=0 and f(e)=0. e is Euler's constant. This space has a special inner product defined as

$$(f,g) = \int (dx/x)\, f^*(x)g(x).$$

Here, the extra factor (1/x) is known as a "weight function" and does not affect the proof of the Completeness Theorem.

Consider an operator A in this space, defined as A{f} = x(d/dx)[x(df/dx)].

1. Show that A is Hermitian.
2. Find the eigenfunctions $\varphi(x)$ and eigenvalues λ, where you may let $\lambda = -\nu^2$, for convenience.

[Hint: use the identity $x^\alpha = \exp(\alpha \ln x)$.]

8.3

Consider the set of functions f(x) in the domain x=[0,L]. Suppose the functions are periodic in L, i.e., f(x+L) = f(x).

(1) Check that the functions f(x) form a vector space under addition and scalar multiplication?
(2) Consider the operator A = i(d/dx), where i = $(-1)^{1/2}$. Prove that this operator is Hermitean. [Careful: the *'s are important in the definition of the inner product.]
(3) Find the complete set of functions associated with this operator.
(4) Check that the eigenvalues are real, and show, by direct integration, that any two distinct eigenfunctions are orthogonal.

8.M Express the function f(x)=1, in the domain x=[0,1], as a series using the complete set sin(nπx). Find the coefficients. Plot truncated series using Mathematica, for truncations with 2 terms, 4 terms, 16 terms, and 32 terms. [If you have experience with Mma, you can use a DO Loop. However, the easiest way to do this is to cut and paste.]