**Phys374/F09/Hassam/PS5** **Due Oct 20, 2009**

**5.1** From Baierlein 1-16

For each of the following forces, test for the existence of a potential-energy function. If such a function exists, construct it (or just exhibit it, having arrived at it by inspired guesswork).

(*a*) $\mathbf{F} = C(x\hat{\mathbf{x}} + y\hat{\mathbf{y}}) \exp[-\alpha(x^2 + y^2)]$.
(*b*) $\mathbf{F} = C(y\hat{\mathbf{x}} - x\hat{\mathbf{y}}) \exp[-\alpha(x^2 + y^2)]$.
(*c*) $\mathbf{F} = C(x\hat{\mathbf{x}} + y\hat{\mathbf{y}}) \exp[-\beta(x^2 + y^2)^{1/2}]$.
(*d*) $\mathbf{F} = C(x^3y\hat{\mathbf{x}} + xy^2\hat{\mathbf{y}} + z\hat{\mathbf{z}})$.

Example: For the force $\mathbf{F} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$, one way to construct a potential energy function is as follows. If $F = -\nabla V$, then $\partial V/\partial x = -x$ and $\partial V/\partial y = -y$. Integrating the first => $V(x,y) = -x^2/2 + f(y)$; plug this into the 2nd eqn => $df/dy = -y$ => $f(y) = -y^2/2 + C$. Thus, $V = -(x^2/2 + y^2/2) + C$. C can be discarded. You can also try to guess the function and then test it.

**5.2**

*This problem is written in "normalized units", ie, the dimensions of various physical quantities may not be directly evident. Normalized units will make the algebra simpler. This problem was also done in a previous problem set but using different methods and different initial conditions.*

A mass m=1 is moving under a central force $\mathbf{F} = -\mathbf{r}$, where $\mathbf{r}(t)$ is the position vector measured from the origin. At t=0, the mass is in the x-y plane at x=1, y=0. The initial velocity is 2, pointing in the y-direction. We will explore the subsequent motion using polar coordinates r(t) and φ(t).

1 Given the initial conditions, what is r(0), (dr/dt)(0), and (dφ/dt)(0)?

2 Write down 2 constants of the motion. Your constant quantities should be evaluated based on the initial conditions. The combinations of variables that stay constant should be explicitly written out in terms of r(t), φ(t), and their time derivatives.

3 Use the constants of the motion in 2 to obtain a decoupled differential equation for r(t).

4 From 3, show that there are certain values of r(t) that are forbidden. Find these values. Make a sketch of a possible particle orbit in the 2D plane.

5. Now solve for x(t) and y(t) in Cartesian geometry directly, using the initial conditions above. Show that the solution for y(x) is an ellipse with major and minor axes consistent with your answer from 4.

**5.3**

A particle of mass m = 1 is moving in a force field **F(x) = -∇**U, where U = xy. x and y are Cartesian coordinates (normalized units). Assume that the motion is 2D only, confined to the x-y plane.

1. Using Newton's Equations, find the coupled ODE's satisfied by x(t) and y(t).

2. Suppose at t=0, we have x(0)=1, y(0)=1, (dx/dt)(0)=0, (dy/dt)(0) = 0. Find x(t) and y(t) for all subsequent t.

3. Identify a constant of the motion. State why this is a constant. Express this constant in terms of x, y, dx/dt, and dy/dt and find its value for the above initial conditions. Plug in your solution from 2 into the expression for the constant and show that indeed this combination stays constant.

4. Show, by direct differentiation, that the combination M = (dx/dt)*(dy/dt) + $(x^2 + y^2)/2$, is also a constant of the motion. ["Direct differentiation" means if M = constant (in time), then clearly dM/dt = 0.] Note: this constant has nothing to do with energy or angular momentum. Find M from the initial conditions in 2 above and plug in your solution from 2 to verify the constancy of M.

5. Suppose we introduce the variables p(t) = x(t) + y(t) and q(t) = x(t) – y(t). Use the equations in <u>1</u> to find the equations satisfied by p and q (ie, your entire new system of equations should only be in terms of p and q). What are the general solutions for p and q? Using the initial conditions above, find p(t) and q(t) and, hence, check your answer above for x(t) and y(t).