Phys374/F09/Hassam/Problem Set 2 **Due 09/22/09**

2.1 Solve for y(x) if y satisfies (here, $y' = dy/dx$)

$$y''' + y'' + y' + y = 0, \qquad y(0) = 0,\ y'(0) = -1,\ y''(0) = 2.$$

2.2 The Van der Pol oscillator is described by the ODE

$$d^2x/dt^2 + x = 2\beta\,(dx/dt)\,[1 - x^2]$$

where $x=x(t)$, and β is a constant. Suppose $x(0) = 0$ and $v(0)=0.1$, where $dx/dt = v$.

(a) is this a linear equation?
(b) Note that at $t=0$, $x=0$. Suppose we are only interested in solving for x(t) for very short times away from t=0. We may then assume that, during this short time, $|x(t)| \ll 1$. If so, what would be a good approximation to the term on the r.h.s of the ODE?
(c) Solve the resulting approximate equation. For this, assume that $\beta \ll 1$ and use this to approximate your answer further.
(d) Make a sketch of your x(t) vs t. Is $|x(t)|$ indeed $\ll 1$ for very early times? Roughly, how far in time would you have to go, in terms of β, for this assumption to break down significantly (do this as a self-consistent check, i.e., demand that $|x(t)| \ll 1$ and insert your solution into this inequality to arrive at the time)?

2.2M Solve the Van der Pol oscillator numerically using NDSolve in Mma (or equivalent). Let $\beta = 1/10$. Make no approximations.

(a) Plot x(t) vs t for {t,0,90}.
(b) What happens to the oscillator in the steady state (for long times)? Can you reason why, physically (think about the type of "friction" that this oscillator feels)?
(c) Use NDSolve to solve the approximate equation you found in 2.1(b) above. Call this x1(t). Plot x1(t) for {t,0,45}. Compare this with x(t) for the complete solution found in 2.1M(a) for the same time period. Do you see agreement for early times? When do the solutions start to diverge, by more than roughly 30%?

2.3 What are the solutions to the equation

$$x(d/dx)[x\ dy/dx] = -k^2y, \quad y = y(x)\ ?$$

Now, try to solve this by another method. Define a new variable s = ln(x), and consider that y = y(s). What is the transformed equation in the s variable? [use the chain rule.] Solve the transformed equation to show that the same solutions can be obtained.

2.4 A mass m is thrown upwards in a gravitational field with acceleration, g, pointing downwards. Let the position of the mass be x(t), and let x(0) = 0, v(0) = v_0. You may set 2 constants to unity in this problem if desired.

(a) find v(t) and x(t) for all subsequent times by direct integration of Newton's equations. Do not use energy conservation. By eliminating t in the expressions for x(t) and v(t), find v(x). Find the time at which the mass comes to rest. Find the maximum height reached.

(b) Now solve this problem using the energy method. Again, find x(t), v(t), and v(x). [making sure that you pick the correct sign of dx/dt at t=0 will suffice to yield the entire x(t).] Make a plot of E and U(x) (as defined in class) and show clearly where the turning point occurs.

2.5 Baierlein 1-11

1-11 This is a one-dimensional problem. The potential energy for the particle of mass m has the symmetric form $U(x) = U_0b^2/(x^2+b^2)$, where U_0 and b are positive constants.

(*a*) Sketch the potential energy as a function of x.

(*b*) If the particle started from far to the left (really, at $x = -\infty$) with an initial speed to the right of $[U_0/(5m)]^{1/2}$, will the particle be able to get to positive values of x? If yes, why? If no, how far to the right will it get?

(*c*) What are the magnitude and direction of the force on the particle when $x = -7$?

2.6 Baierlein 1-22 (attached)

1-22 An object of mass m, initially at rest at location $x = x_0$, is attracted to the origin by a force of magnitude C/x^3. Determine the time it takes the object to reach the origin. (You do *not* need to solve directly a second-order differential equation. See what energy conservation will do for you. Also, try dimensional analysis: what combination of m, C, and x_0 has the dimensions of time?)