Phys374/F09/Hassam/Problem Set 1 **Due 09/15/09**

1.0 Newton1 watches the apple fall and figures there must be an attractive force F between any mass m and the Earth mass M. He guesses that the force must have the form $F = GMm/r^n$, where G and n are as yet unknown constants and r is the position radius of the mass m. He then proceeds to deduce GM and n as follows:

(a) He measures the acceleration during the fall and finds it be g = 10 m/s2. Using his own other equation (F=ma), he finds a relationship between GM and n. What is this relationship? (Assume that during the fall, the radius of the apple from the Earth center is very nearly R_E, the radius of the Earth.)

(b) He now applies this new force to the circular motion of the moon. Assuming that he knows the radius of the moon orbit, R_M, and its period of rotation around the Earth, T, find another relationship between GM and n.

(c) Using the above two relationships, deduce n (use the known values of T, R_E, and R_M). Assuming this must be an integer, round off and fix n. Thus, find GM.

1.1 A sphere of mass m is placed, at rest, in molasses at t=0. A time dependent force, $F(t) = F_0(t/\tau)$, is applied to the sphere at t=0. The friction force is $-m\beta v$, where v(t) is the velocity. Find v(t). There is no gravity. [Note: if you use the "homogenous + particular solution" method, note that the particular solution is not straightforward to find; though it can be found.]

1.1M Solve for v(t) from 1.1 using Mathematica. Let m=1, β=1, F_0/τ=1:

(a) analytically using DSolve
(b) numerically using NDSolve

In each case, plot the solution from t=0 to t=3, and compare.

1.2 Solve for y(x) if y satisfies

$$dy/dx - y = \sin(x), \quad y(0) = 0.$$

[the identity 2i sin(x) = exp(ix) – exp(-ix) may be helpful to do integrals]

1.3 Find a solution for y(x) where y satisfies

$$dy/dx - y^2 = 1$$

[you may use integral formulae/tables]

1.4 A mass, m, falls under a gravitational acceleration, g. The mass starts from rest at time $t = t_0$. There is a time-dependent retarding friction force, $f = m\, v\, b\, (t_0/t)^{1/2}$, where b = constant and v(t) is the velocity.

(a) What do you expect for the speed v(t) as $t \to \infty$? Prove this by a self-consistent check, i.e., show that for $t >> t_0$, the term you neglected is very small compared to a term you kept.

(b) Solve for v(t).