Equilibrium sailing velocities

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The equations of motion of a sailboat have been analytically solved for motion parallel to the wind velocity. For this point of sailing, the equilibrium velocity and acceleration time have been determined as a function of boat parameters and wind velocity. For other sailing angles, numerical solutions of the equilibrium equations give the conditions for achieving maximum speed while numerical integrations of the equations of motion give the acceleration time.

I. INTRODUCTION

Sailing is an activity enjoyed by many physics students and faculty. Despite the fact that some aspects of sailing can be discussed on a fundamental physics level, providing, for example, an analytically solvable nonlinear force problem, there is little mention of sailing in the physics literature. In addition to purely didactic reasons for the study of
the physics of sailing, there are some aspects of sailing that could benefit from an application of first principles. For example, the forces responsible for yacht motion are given in terms of coefficients that are either experimentally determined, numerically calculated from idealized models, or obtained by analogy with the effects of flow over simple objects such as cylinders and plates. Generation of turbulence and waves is present but difficult to include in the transfer of momentum and energy between the yacht and its environment. A better understanding of these effects is important not only for its own sake but could also lead to improved boat and sail design.

In this article, we consider a more elementary problem, the dynamics of a sailboat assuming the simplest realistic form for the nonlinear velocity dependence of the forces on a boat. For the case of a downwind run (sailing in the direction of the ambient air velocity), we find an analytic solution to the equation of motion from which we determine the characteristic time for acceleration and the equilibrium velocity. The general conditions for equilibrium sailing at any angle are given and the equilibrium velocity is calculated for different course directions with respect to the wind velocity and compared with the analytic solution and numerical integrations of the equations of motion. The conditions are then found for which maximum equilibrium speed is obtained.

II. EQUATIONS OF MOTION

The most relevant forces acting on a sailboat are fluid dynamical forces of two types, aerodynamic and hydrodynamic. In the simplest approximation, both forces depend on the so-called apparent fluid velocity \( \mathbf{V}_a \), which is the velocity of the fluid in the yacht rest frame. The technique of sailing is often difficult for the novice because the apparent velocity, which is involved in propelling the boat forward, depends on the boat velocity \( \mathbf{V}_b \) as well as the fluid velocity \( \mathbf{V}_a \), measured in a frame of reference attached to the Earth’s solid surface. This latter velocity, in the case of air, is often called the “true” velocity in the sailing literature. Despite the fact that this name is unphysical, we will use it rather than invent a new term. The “true” velocity is the vector sum of the apparent velocity and the boat velocity as illustrated in Fig. 1.

It is traditional and useful to decompose the fluid forces into two components called lift and drag.\(^{1,4}\) The drag component \( D \) is parallel to the apparent velocity, and the lift component \( L \) is perpendicular to the apparent velocity. Both components can be written in terms of dimensionless coefficients \( (C_L, C_D) \) and the kinetic energy density of the incoming fluid in the yacht rest frame,

\[
L = C_L \rho v^2 A, \quad D = C_D \rho v^2 A,
\]

where \( A \) is a projected area of the foil (e.g., sail or keel) and \( \rho \) is the fluid mass density. We will take \( A \) to be the maximum projected area (planform area) of the sail or keel–hull combination. In general, the coefficients depend on the Reynolds number of the flow; however, for the Reynolds numbers characteristic of sailing, they are approximately constant. Consequently, we will here consider the coefficients to be independent of the Reynolds number. The coefficients do depend on the attack angle, which is the angle made by the far upstream apparent fluid velocity vector with respect to the foil (Fig. 2). The angular dependence takes into account changing flow velocity on the foil surface, separation of the flow from the foil leading to vortices and turbulence, as well as the changing projected area. In Fig. 3, we show the dependence of \( C_L \) and \( C_D \) on attack angle for a typical, though imaginary, sail and keel.\(^{1,3}\) The coefficients are usually determined either by analytical theory, model experiments, computer simulation, or a combination of methods. While the present state of none of these methods is capable of giving accurate values representative of a wide range of real sailing conditions, the qualitative behavior is expected to be similar to what is shown in Fig. 3. In the numerical calculations that follow, we will use the coefficients shown in the figure by expressing the coefficients as a fourth-order polynomial in the attack angle.

Above the water surface, the aerodynamic forces \( (L_a,\)

- Fig. 1. The vector relationship between the yacht velocity vector \( \mathbf{V} \), the apparent \( (\mathbf{V}_a) \), and “true” fluid velocity \( (\mathbf{V}_t) \) vectors as defined in the text. The angles \( \alpha, \beta, \) and \( \gamma \) are measured with respect to an arbitrary, two-dimensional \((x,\nu)\) coordinate system attached to the Earth’s solid surface. In general, these relationships are true both above and below the air–water surface.

- Fig. 2. (a) Aerodynamic and (b) hydrodynamic attack angles. Here, \( \mathbf{V}_{a\alpha} \) and \( \mathbf{V}_{a\beta} \) are, respectively, the aerodynamic and hydrodynamic apparent fluid velocities.
$D_x$ primarily act on the sails while below the surface the hydrodynamic forces ($L_x, D_x$) act on the submerged keel-hull. In addition, water waves generated on the surface contribute to the force on the hull. In this article we will ignore boat-generated as well as other surface waves. We will also consider a yacht with a single keel and sail, or with an equivalent fictitious sail replacing several actual sails. Initially, we will consider that the foils remain in a plane perpendicular to the water surface, i.e., we neglect the effects of heeling. We will later relax this requirement and consider the effects of heeling. Finally, the yacht is considered to be tuned so that the center of forces above and below the water surface is located on a line passing through the center of mass, thereby eliminating the necessity to consider torques causing rotation about the other two axes.

The aerodynamic attack angle for the sails $\theta_s$ is controlled by adjusting the angle of the sail boom $\theta_{sb}$ with respect to the centerline of the boat. We will denote $\theta_{cb}$ the angle of the boat centerline with respect to the same $(x,y)$ coordinate system in which the angle of the apparent wind velocity is $\beta$. Referring to Fig. 4 we see that the aerodynamic attack angle is

$$\theta_s = \beta - \theta_{cb} - \theta_{sb}. $$

The attack angle for the keel is somewhat simpler since we will be considering still water, i.e., zero "true" water velocity. The hydrodynamic apparent fluid velocity is then just the negative of the boat velocity. In this case, the hydrodynamic attack angle $\theta_h$ is the difference between the orientation of the boat centerline and the yacht velocity vector $\Gamma$ in Fig. 4,

$$\theta_h = \theta_{cb} - \Gamma. $$

In order to apply Newton's second law to a sailing yacht, it is convenient to use a two-dimensional coordinate system in the plane of the air–water interface. We will use the arbitrarily defined $(x,y)$ system shown in Fig. 4, where, above the surface, the apparent wind velocity vector makes an angle $\beta$ with the $x$ axis and the yacht velocity vector angle is $\Gamma$. The orientation of the vector components is shown in Fig. 5. The two components of the equation of motion are

$$m \frac{dV_x}{dt} = L_a \sin(\beta) - D_a \cos(\beta) - L_h \sin(\Gamma) - D_h \cos(\Gamma),$$

$$m \frac{dV_y}{dt} = -L_a \cos(\beta) - D_a \sin(\beta) + L_h \cos(\Gamma) - D_h \sin(\Gamma).$$

The arbitrariness of the coordinate system can be appreciated by considering the components of the force vector parallel and perpendicular to the instantaneous yacht velocity vector:

$$F_x = F_x \cos(\Gamma) + F_y \sin(\Gamma) = L_a \sin(\beta - \Gamma) - D_a \cos(\beta - \Gamma) - D_h,$$

$$F_y = -F_x \sin(\Gamma) + F_y \cos(\Gamma) = L_a \cos(\beta - \Gamma) + D_a \sin(\beta - \Gamma) - L_h.$$
The angle \((\beta - \Gamma)\) is called the apparent wind heading angle. We see that only the coordinate independent relative heading angle is important.

Because we are considering still water, the apparent fluid velocity appearing in the hydrodynamic forces \(D_a\) and \(L_a\) as stated above, is just the negative of the yacht velocity, whose magnitude can be related to the "true" air speed \(V_{a,a}\), by using the law of sines on the vector triangle in Fig. 1,

\[
V = V_{a,a} \sin(\alpha - \beta) / \sin(\beta - \Gamma).
\]

The aerodynamic forces \(L_a\) and \(D_a\) depend on the apparent air velocity \(V_{a,a}\), the magnitude of which can also be expressed in terms of \(V_{a,a}\) as,

\[
V_{a,a} = V_{a,a} \sin(\alpha - \Gamma) / \sin(\beta - \Gamma).
\]

In general, the forces are more nonlinear than quadratic since \(\beta\) also depends on the yacht velocity. Finally, we note that in the equation of motion the mass \(m\) is the total mass accelerated. Since the lift and drag coefficients are usually only determined for steady flow conditions, the nonsteady conditions existing during acceleration can be accommodated by making \(m\) slightly larger during acceleration to include the air and water accelerated with the boat. This is a small effect, which we ignore.

### III. DOWNWIND RUN

The equations of motion can be analytically solved in the simplest case of a downwind run in still water because the force simplifies to a quadratic dependence on speed. For the arbitrary \((x,y)\) coordinate system shown in Fig. 6, we have taken the "true" wind direction to be the negative \(y\) direction so that the angle \(\alpha\) is 90°. For a downwind run in still water, the keel centerline is aligned with the yacht velocity vector, which means that the hydrodynamic attack angle \((\theta_a)\) is zero. In the notation of Fig. 4, \(\theta_a = \Gamma\) and, hence, for downwind sailing with \(\alpha = 90°\), this puts \(\beta\) and \(\Gamma\) equal to 270°. A further simplification results from the fact that for a downwind run the sail boom is oriented at a right angle with respect to the centerline of the boat \((\theta_{bo} = 90°)\), which makes the aerodynamic attack angle \(\theta_a\) equal to 90°. Because of the values of the attack angles and the fact that they remain constant, the lift coefficients are zero and the drag coefficients are nonzero and constant. This means that there are no perpendicularly forces, i.e., \(F_{\perp} = 0\). Rewriting the \(y\) equation of motion, dropping the \(x\) and \(y\) notation \((V_x = V, V_y = 0)\), we have

\[
\frac{dV}{dt} = K_o \left( V - V_{a,a} \right)^2 - K_v V^2,
\]

where the two constants are given by \(K_o = \frac{1}{2} \rho_c C_{a,b} A_{s} m\) and \(K_v = \frac{1}{2} \rho_c C_{a,d} A_{s} / m\). For a yacht starting from rest with constant "true" wind speed, the yawning velocity at later times is found by integrating the equation of motion finding

\[
V = V_{a,a} \left[ \frac{1}{1 + (K_o/K_v)^{1/2}} \right] \left( \frac{1}{1 + (K_o/K_v)^{1/2}} \right),
\]

where \(\tau^{-1} = (K_o/K_v)^{1/2} V_{a,a}\). As \(\tau \to \infty\), the yacht speed approaches the downwind equilibrium speed,

\[
V(\infty) = V_{a,a} \left[ \frac{1}{1 + (K_o/K_v)^{1/2}} \right].
\]

For a hull with negligible drag \((K_o \approx 0)\), the downwind equilibrium speed is a maximum and approaches the "true" wind speed, while in the opposite extreme of sufficiently large hull drag the minimum equilibrium speed of \(V_{a,a}(K_o/K_v)^{1/2}\) is obtained. The magnitude of the \(K\) ratio may be estimated for typical parameters. With \(\rho_c/\rho_g \approx 1000, C_{a,b}(0°) \approx 0.01 - 0.1, \) and \(C_{a,d}(90°) \approx 1\), we find that \(10 A_s/A_{s} < K_o/K_v < 100 A_s/A_{s}\). For the boat parameters used here, \(K_o/K_v \approx 6\), giving a normalized downwind speed of about 0.3.

A widely used figure of merit for a sailboat's potential as a racing boat is the ratio of displacement to sail area \((W/A_s)\), often expressed in the US in tons/100 ft\(^2\). One might question why the displacement should be a factor in the equilibrium speed. Looking at the ratio \(K_o/K_v\), one can see how this comes about. Since the area \(A_s\) is the underwater projected hull and keel area, it can be multiplied by an effective width \(w_h\) to give the displacement \(W = \rho_c g A_{s} w_h\). The \(K\) ratio is then seen to contain the displacement to sail area ratio, \(K_o/K_v = (C_{a,b}/C_{a,d}) (W/A_s) w_h \rho_c g\). Hence, for a given average hull width, the equilibrium speed depends inversely on the displacement to sail area ratio. The mass, of course, directly affects the acceleration time required to reach some fraction of the

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*Fig. 7. Yacht speed as a function of time for a downwind run for different mass yachts with \(A_s = 23.2\) m\(^2\) and \(A_{s} = 1.86\) m\(^2\) with \(V_{a,a} = 3\) m/s. From top to bottom the masses are 730, 1459, 2189, and 2919 kg. The figures of merit \((W/A_s)\) are 0.32, 0.64, 0.96, and 1.28 tons/100 ft\(^2\).*

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equilibrium speed. For $K_s/K_a \gg 0.01$, the time to reach 90% of $V(\infty)$ is approximately

$$\tau_s = 1.5/[ (K_s/K_a)^{1/2} V_{,s} ] .$$

While this time depends on the displacement and sail area, it does not have the $W/A_s$ functional dependence.

In Fig. 7, we compare the yacht speed as a function of time for yachts with four different masses and the same sail area (250 ft$^2$, 23.2 m$^2$). As expected from inspection of the equations, the acceleration times vary inversely with the mass and the equilibrium velocities are the same for all four yacht masses. For the heaviest yacht (3200 lb, 1459 kg), and for $W/A_s = 1.28$ tons/ft$^2$, the acceleration time is on the order of 60 s and the equilibrium speed is about 0.3 $V_{,s}$.

The velocity as a function of time for different sail areas and fixed mass is shown in Fig. 8. For this comparison, both the acceleration time and the equilibrium speed change.

IV. EQUILIBRIUM VELOCITIES

For other sailing angles, it is not so easy to find an analytic expression for the yacht velocity as a function of time. We will first find the equilibrium conditions (i.e., when $dV/dt = 0$) at arbitrary sailing angles and then compare the resulting equilibrium speeds with numerical integrations of the equations of motion. According to Marchaj, it was shown by F. W. Lancashire in 1907 that, for a constant velocity yacht, the equilibrium heading angle into the apparent wind is the sum of the drag angles, i.e.,

$$\beta - \Gamma = \epsilon_a + \epsilon_h ,$$

where the drag angles are defined as $\epsilon = \tan^{-1}(C_d/C_I)$. This is just the condition that the hydrodynamic and aerodynamic forces be collinear, and it is obtained by putting the lhs of the equations of motion equal to zero. Notice that this condition contains no information on the absolute strengths of the forces.

The relation between the sum of the drag angles and the apparent wind heading angle is, however, only a necessary condition for equilibrium. In addition, for the net force to be zero, the magnitude of the hydrodynamic and aerodynamic forces must also be equal, i.e.,

$$L_a^2 + D_a^2 = L_k^2 + D_k^2 .$$

This condition does contain information about the absolute forces through the sail and hull areas and hence is in some sense more boat design dependent than the angle relation.

At this point, we can show that for equilibrium sailing of a particular yacht we have four more variables than equations. By specifying two variables, we can find the velocity ratio $V/V_{,s}$ as a function of one of the others.

Because of the complexity of the equations (resulting from the polynomial expansion for the drag and lift coefficients) it is not convenient to combine the equilibrium conditions and the velocity triangle equations to find a single equation for the equilibrium velocity. As a practical matter, the equations must be solved numerically.

One methodology for sailing is to steer the boat so that its centerline is pointing in a certain direction with respect to the “true” wind and adjust the angle of the sail boom with respect to the centerline of the boat to maximize the yacht velocity. In the analysis of this situation, we specify $\sigma$ and $\theta_{sb}$, vary $\theta_{sb}$, and solve for $V/V_{,s}$. In Fig. 9, we plot the normalized equilibrium speed of the boat as a function of...
\( \theta_{eq} \) with \( \theta_{eq} = 44^\circ \) and \( \alpha = 90^\circ \). We see that the equilibrium speed is a maximum for \( \theta_{eq} \approx 20^\circ \) and that the maximum equilibrium speed is achieved near (but not equal to) the minimum value of \( \beta \); i.e., the largest angle between the “true” wind direction and the apparent wind direction for which equilibrium is possible at the fixed value of \( \theta_{eq} \). Because of the small variation in \( \Gamma \), the maximum speed is also near the minimum apparent heading angle. Attempts to find a simple algebraic relationship between the maximum speed and the sail boom angle were not successful.

The equilibrium calculation was repeated for many different centerline angles \( \theta_{eq} \) and the maximum speed at each angle was determined and is plotted as a function of the centerline direction in Fig. 10. We see that the maximum speed occurs for \( \theta_{eq} \approx 0^\circ \), i.e., at 90° wrt the “true” wind velocity. A quantity of interest to sailors is the “speed made good,” \( V_{mg} \), which is defined as the component of boat velocity along the direction the “true” wind is coming from. This quantity is strongly dependent on the functional dependence of the lift coefficient on angle for small and medium attack angles. For the yacht considered here, the maximum “speed made good” occurs for \( \alpha - \theta_{eq} \approx 40^\circ \). The equilibrium velocity becomes zero for some value of \( \alpha - \theta_{eq} \) and negative for even smaller angles.

In order to verify that the equilibrium velocities are accessible and stable, the time-dependent equations of motion were numerically integrated, using a fourth-order Runge–Kutta method, until the velocity reached an equilibrium value. Equilibrium was defined to be when the normalized speed ratio \( V/V_{na} \) changed by less than 0.001 in 2 s. The sail boom angle was varied and the normalized equilibrium speed as well as other variables were determined as a function of \( \theta_{eq} \). For each sail angle, the yacht reaches a particular speed regardless of whether its initial speed is above or below the equilibrium value for that \( \theta_{eq} \). The equilibrium speeds reached are consistent with the equilibrium values shown in Figs. 9 and 10. The yacht speed approaches its equilibrium value with a dependence similar to the hyperbolic dependence of the downwind run. In Fig. 11, the velocity as a function of time for \( \theta_{eq} = 44^\circ \) and \( \theta_{eq} = 23^\circ \) is shown and compared with the curves of

\[
V = V_{na} \left[ 1 + K_s \coth(t/\tau_s) \right],
\]

where \( K_s \) is chosen to give the correct asymptotic value \( (K_s = 1.4184) \) and with two values of \( \tau_s \) (15 and 20 s). At late times \( (t > 10 s) \), when the velocity vector orientation has become constant, the time constant appears to be about 16 s. This compares with the analytically determined time constant of 12.8 s for the same yacht in a downwind run. The magnitude of the aerodynamic and hydrodynamic forces initially varies widely but after about 10 s the force magnitude equilibration condition is satisfied and later the force vector angle collinearity condition is fulfilled.

V. HEELING EFFECTS

In addition to the center of mass motion, various torques cause rotation about the center of mass. We will, however, only consider rotation about an axis through the center of mass and parallel to the centerline (called heeling).

Often hulls are designed so that the center of effort of the buoyant force \( (F_b) \) moves as the boat heels so as to increase the lever arm. Our calculations will not include this effect but instead will be based on a cylindrical hull cross section where the buoyant force acts on a fixed point.

The torque about this axis has three contributions, from the aerodynamic force, the hydrodynamic force, and the buoyant force. Referring to Figs. 5 and 12, we find for the three torques:

\[
T_b = H_b \left[ L_b \cos(\beta - \theta_{eq}) \right] + D_b \sin(\beta - \theta_{eq}) \cos \theta,
\]

\[
T_o = H_o \left[ L_o \cos(\Gamma - \theta_{eq}) \right] + D_o \sin(\Gamma - \theta_{eq}) \cos \theta,
\]

\[
T_o = -H_b F_b \sin \theta.
\]

The rotational and vertical motions are strongly affected by hull drag and quickly come into equilibrium. To simplify calculations, we assume that static angular and vertical equilibrium exists at all times. The buoyant force is then
determined by the vertical force balance,
\[ F_v = mg + (L_u - L_v) \sin \theta. \]
For a sufficiently massive boat and/or small angles of heel, the buoyant force is to a good approximation given by the total weight. Using these approximations, the heel angle is given by
\[ \cos \theta = \frac{(T_h - T_D - T_L) (T_b^2 + T_D^2 - T_L)^{1/2}}{(T_b^2 + T_D^2)}, \]
where
\[ T_h = H_u L_u \cos (\beta - \theta_4), \]
\[ T_D = H_u L_u \sin (\beta - \theta_4), \]
\[ T_L = H_u L_u \cos (\Gamma - \theta_4). \]

The angle of heel affects the center of mass motion along the water's surface because the horizontal components of the lift forces are reduced to \( L_u \cos \theta \) and \( L_u \sin \theta \). Incorporating this effect into heel into the equations of motion of the center of mass we have integrated the equations to find the resulting motion for the same centerline angle as in the no heel case but with the sail adjusted to give the maximum equilibrium speed. The maximum equilibrium speed was not determined from an enlarged set of equilibrium equations but instead from many integrations of the equations of motion. In Fig. 11 we show how the speed evolves in time for a particular set of boat parameters chosen to give a reasonably large heel angle (\( H_u = 4.57 \text{ m} \), \( H_D = 1.52 \text{ m} \), and \( H_L = 0.15 \text{ m} \)). In these calculations, the angle changes as the speed increases but is typically on the order of 30°. Again, comparing with a hyperbolic cotangent dependence, the time constant appears to be slightly less (15 s) than in the no heel situation and, of course, the equilibrium speed is less. As before, we notice that the magnitude of the aerodynamic and hydrodynamic forces rapidly becomes quite close, while on a slower time scale the angle of the force vectors becomes aligned.

The effects of heel are complicated by the fact that the relevant torques are speed and angle dependent as well as dependent on the boat design. The calculations performed here are hypothetical in several important ways. Not only is the model hull design more simplified than is the case for actual boats but, for the runs calculated, the yacht centerline angle was held fixed, which is not usually the best way to sail. While it appears that for the model yacht considered here heeling drastically reduces speed, one could imagine hulls for which this either is not the case or at least has a different dependence. Because of these complications, the more detailed effects of heeling were not investigated here.

VI. CONCLUSIONS

The present work tends to justify and extend some aspects of conventional sailing wisdom that have for the most part evolved empirically. Sailors with limited equipment have a limited amount of information on which to base their decisions on how to sail. Many small boat racing sailors equip their boats with an indicator on top of their masts to show the direction of the apparent wind velocity.

We have found that to achieve maximum speed for a given boat pointing angle (\( \alpha - \theta_4 \)), see Figs. 1 and 4), it is reasonable to maximize the angle between the incoming apparent wind velocity vector and the “true” wind velocity vector (\( \alpha - \theta_4 \)). This is difficult to do since the sailor generally does not know precisely the “true” wind direction. For sailing toward the wind source, one wants to maximize \( V \cos (\alpha - \Gamma) \), the so-called “speed made good.” Guidelines exist for setting the sail boom for upwind sailing.

From our calculations one sees that the maximum “speed made good” will occur at a pointing angle of about 40°. For this pointing angle, the angle of the apparent wind with respect to the centerline of the boat (\( \beta - \theta_4 \)) is 29°, which is comparable to a suggested value of about 30°. It should be emphasized that the exact angle depends on details of the sail design and sail cut as well as the hull and keel design, both of which affect the lift and drag coefficients.

Another topic this work touches on has to do with the relevance of the minimum total drag angle, \( \epsilon_1 + \epsilon_2 \), in determining maximum speed upwind. For the lift and drag coefficients used here, the minimum value of \( \epsilon_1 \) is 6.7°, which occurs at an attack angle of 8°, while the maximum of \( \epsilon_1 \) is 11.6 at 9°. If it were possible to sail with these attack angles, the total drag angle, and hence the apparent heading angle for equilibrium, would be 18.3°. We have found that for every pointing angle the maximum equilibrium boat velocity is achieved near the minimum, possible total drag angle for that pointing angle, which is generally larger than 18.3°. For a range of pointing angles from 10° to 180°, the total drag angle varied from about 18° to 180°, respectively. As we can see in Fig. 10, the maximum speed made good occurs at a total drag angle of 31°. Clearly, reducing the drag angle is important for upwind sailing; however, using the minimum possible drag angle in the velocity triangle relation does not determine the maximum upwind speed.

5. J. Baader, *The Sailing Yacht* (Norton, New York, 1979), p. 59. The behavior of the lift at small attack angles is dependent on details of the sail cut, tension, and battens. The sail behavior chosen in this article, with residual lift at zero attack, is not common but is seen in small catamarans.
6. Reference 1, p. 564.