Problem set 6

1. Consider the BCS Ansatz for the many-body wave-function of a superconductor
$$|\mathrm{BCS}>=\prod_{\mathbf{k}}\left(u_{\mathbf{k}}+v_{\mathbf{k}}c^{\dagger}_{\mathbf{k}\uparrow}c^{\dagger}_{-\mathbf{k}\downarrow}\right)|\mathrm{vac}> .$$
Using this wave-function calculate the average number of particles $\langle\hat{N}\rangle$ and the fluctuations in the number of particles $\left\langle\left(\hat{N}-\langle\hat{N}\rangle\right)\right\rangle$

2. Calculate the difference in energy between the BCS state and the Fermi gas state (express your result in terms of the superconducting gap Δ).

3. At finite temperature, the self-consistency equation for the superconducting gap has the following form (see lectures):
$$\Delta_{\mathbf{k}}=-\sum\frac{V_{\mathbf{kl}}\Delta_{\mathbf{l}}}{2E_{\mathbf{l}}}\left[1-2f(E_{\mathbf{l}})\right],$$
where $E_{\mathbf{k}}=\sqrt{\xi_{\mathbf{k}}^2+\Delta_{\mathbf{k}}^2}$ and $f(E)$ is the Fermi-Dirac distribution function (corresponding to the Bogoliubov quasiparticles). Consider the case of zero-range attractive potential and find the superconducting transition temperature, T_{c}. Find the asymptotic behavior of the superconducting gap near the transition $T_{\mathrm{c}}-T\ll T_{\mathrm{c}}$.

Due Thursday, April 12 (in class)