Supplemental material for "Early-Time Instabilities in Non-Chaotic Quantum Systems"

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Quantum dynamics in integrable polygonal billiards and a 1D particle in a box — We show here that integrable polygonal billiards and even the textbook example of a particle in a 1D box also demonstrate what appears as an exponential instability in the OTOC. Fig. S1 shows fast growth of the OTOC by more than 8 orders of magnitude in a rectangular billiard. As this billiard is completely separable in the (x, y) coordinates, and we only consider operators along the x axis, we should observe exactly the same dynamics in a 1D particle-in-a-box model. We do an independent semi-analytic calculation of the OTOC in the latter case. In particular, we use the analytically known eigenvalues and eigenfunctions of the 1D box to decompose the initial Gaussian wave packet, $\Psi_0 = \sum_n \psi_n |n\rangle$, where $\psi_n = \langle n|\Psi_0\rangle$ and $\hat{H}_{\text{box}}|n\rangle = E_n|n\rangle \equiv \hbar_{\text{eff}}\varepsilon_n|n\rangle$, and propagate it in time. The only two steps done numerically are the calculation of the overlap coefficients ψ_n and the summation of the corresponding series, as shown in the following equations:

$$C(t) = -\left\langle \Psi_0 \left| [\hat{x}(t), \hat{p}_x(0)]^2 \right| \Psi_0 \right\rangle = -\sum_{n,m} \psi_n^* \psi_m \left\langle n \left| [\hat{x}(t), \hat{p}_x(0)]^2 \right| m \right\rangle, \tag{S1}$$

$$\left\langle n \left| [\hat{x}(t), \hat{p}_x(0)]^2 \right| m \right\rangle = \sum_k \left\langle n \left| [\hat{x}(t), \hat{p}_x(0)] \right| k \right\rangle \left\langle k \left| [\hat{x}(t), \hat{p}_x(0)] \right| m \right\rangle, \tag{S2}$$

$$\left\langle n \left| \hat{x}(t)\hat{p}_x(0) \right| k \right\rangle = \left\langle n \left| e^{it\frac{\hat{H}_{\text{box}}}{\hbar_{\text{eff}}}} \hat{x} e^{-it\frac{\hat{H}_{\text{box}}}{\hbar_{\text{eff}}}} \hat{p}_x \right| k \right\rangle = \sum_j e^{it\varepsilon_n} (x)_{nj} e^{-it\varepsilon_j} (p_x)_{jk}, \tag{S3}$$

$$(x)_{nm} = \left(\frac{2}{\pi}\right)^2 L \frac{nm\left[(-1)^{n+m} - 1\right]}{\left(n^2 - m^2\right)^2}, \quad (p_x)_{nm} = \frac{2}{L} \frac{nm\left[(-1)^{n+m} - 1\right]}{\left(n^2 - m^2\right)}, \quad n \neq m, \tag{S4}$$

where L is the length of the box, $(x)_{nm}$ and $(p_x)_{nm}$ are the matrices of the operators $\hat{x} = \hat{x}(0)$ and $\hat{p}_x = \hat{p}_x(0)$, respectively, in the eigenbasis of the model. $(x)_{nn} \equiv L/2$ and $(p_x)_{nn} \equiv 0$ for any n. We repeatedly used the resolution of identity in these transformations. And indeed, we obtain the same results as in our numerical curves for the rectangular billiard that show the same exponential-looking growth – see Fig. S1. Given that this system is 1D and integrable, its dynamics is inevitably periodic, which is confirmed by extending the time further, as demonstrated in Fig. S2. However, at relatively short times, the growth of OTOC and the L-correlator both show divergences well fitted by exponential functions.

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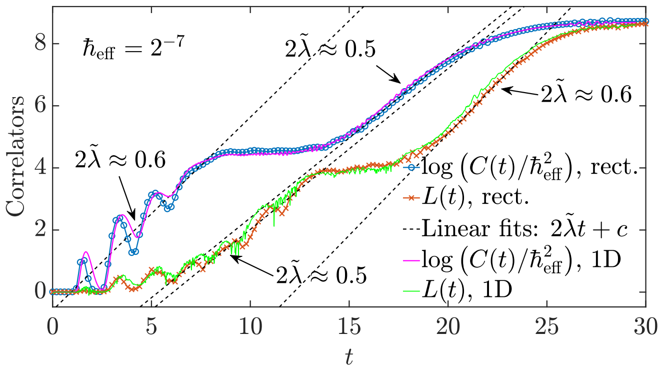

FIG. S1. Logarithm of the OTOC, $\ln(C(t)/\hbar_{\rm eff}^2)$, in a rectangular billiard (upper blue line and open circles). The OTOC shows periods of what appears as exponential growth superimposed with oscillations due to collisions. The growth rate is the smallest among our examples but can be made continuous all the way to the saturation value by varying the initial conditions. The related L-correlator, introduced in Fig. 5 in the main text, is shown for comparison (lower red line and crosses) and demonstrates the same behavior. Black dashed lines show linear fits with $2\tilde{\lambda} \approx 0.5 \div 0.6$, which is over 3 times larger than the individual inverse time-windows, ensuring the adequate fit. Pink and light-green lines show the comparison with the case of a particle in a 1D box. The data agrees, as expected. Longer-time data for the 1D case is presented in Fig. S2.

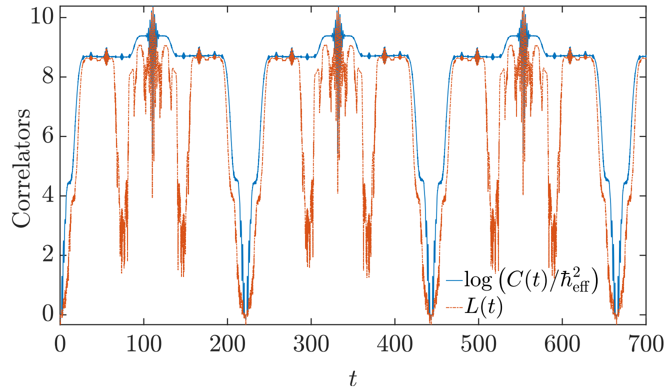

FIG. S2. Logarithm of the OTOC, $\ln(C(t)/\hbar_{\rm eff}^2)$, for a particle in a 1D box (upper blue line). The OTOC shows periods of “exponential” growth and decay, as well as those of oscillatory and stable behavior, and its evolution is strictly periodic. The L-correlator, introduced in Fig. 5 in the main text, is shown for comparison (lower red dashed-dotted line). It demonstrates the same “exponential” instability at early time and periodic structure at longer time. The length of the box of ≈ 1.65 and the particle's momentum $p_0 \approx 0.89$ are chosen to correspond to those along the $x-$axis in the rectangular billiard in Fig. S1.