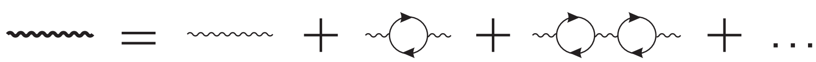
Homework #2 — PHYS 625 — Spring 2018 Professor Victor Galitski
Deadline: Wednesday, May 2, 2017, in class Office: 2270 PSC

Web page: `http://terpconnect.umd.edu/~galitski/PHYS625/`

Do not forget to write your name and the homework number!

Screening. Phonons.

1. The screening of Coulomb interaction in an electron gas is described by the following diagram series,



where the thin wavy line is the Coulomb interaction and the bubble corresponds to the polarization operator

$$\Pi(\omega, \mathbf{q}) = 2i \int G\left(\varepsilon + \frac{\omega}{2}, \mathbf{p} + \frac{\mathbf{q}}{2}\right) G\left(\varepsilon - \frac{\omega}{2}, \mathbf{p} - \frac{\mathbf{q}}{2}\right) \frac{d^d\mathbf{p}}{(2\pi)^d} \frac{d\varepsilon}{2\pi},$$

where $G(\varepsilon, \mathbf{p})$ is the free electron Green's function and d is the dimensionality of the system.

The dynamically screened Coulomb interaction is described by the following formula

$$V(\omega, \mathbf{q}) = \frac{v(\mathbf{q})}{1 - v(\mathbf{q})\Pi(\omega, \mathbf{q})},$$

where $v(\mathbf{q})$ is the bare Coulomb potential.

Consider a *two-dimensional* ($d = 2$) electron gas and...

(a) Calculate the bare (unscreened) Coulomb interaction in momentum space, by Fourier-transforming the potential $v(r) = e^2/r$.

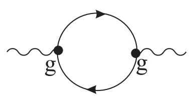
(b) Calculate the polarization operator at $\omega = 0$ and $\mathbf{q} = \mathbf{0}$, and find the analog of the Debye screening length in two dimensions (write the screened Coulomb potential at $q = 0$ and $\omega = 0$).

(c) Calculate the screened Coulomb potential in real space.

(d) Calculate the polarization operator at small but finite frequencies and momenta $\omega \ll E_{\mathrm{F}}$ and $q \ll p_{\mathrm{F}}$. Find the spectrum of collective modes (two-dimensional plasmons). This spectrum is the solution of the equation $v(\mathbf{q})\Pi\left[\omega(\mathbf{q}), \mathbf{q}\right] = 1$. What is the main difference between the two-dimensional and three-dimensional plasmons?

2. Phonon excitations in a crystal are often described by the Debye model in which the phonon spectrum is $\omega(\mathbf{k}) = c|\mathbf{k}|$, if $k < k_{\rm D}$ (where $\omega_{\rm D}$ is the Debye frequency) and $\omega(k) = 0$, if $k > k_{\rm D}$. Find the threshold Debye frequency in a crystal of volume V, which contains $N \gg 1$ identical atoms. Assume that only longitudinal phonons are present.

3. The leading order correction to the phonon propagator is given by the diagram (*c.f.*, problem 3 above),



$$\Longrightarrow \quad D^{-1}(\omega,\mathbf{k}) = D_0^{-1}(\omega,\mathbf{k}) + g^2\,\Pi(\omega,\mathbf{k})$$

where the wavy line corresponds to the phonon propagator (use the Debye model), the expression for the bubble is given above, and each vertex corresponds to the electron-phonon coupling constant g.

Find the leading order correction to the speed of sound in three dimensions. Assume that the bare speed of sound is much smaller than the Fermi velocity, $c \ll v_{\rm F}$.

Reading: Abrikosov, Gor'kov, and Dzyaloshinskii and Lectures
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