Homework #1 — PHYS 625 — Spring 2018
Deadline: Wednesday, February-7, 2017, in class

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Do not forget to write your name and the homework number!

Bogoliubov transformation

1. Consider a classical chain of oscillators (see also your lecture notes)

$$\mathcal{H} = \sum_{n=-\infty}^{+\infty} \left[\frac{p_n^2}{2m_n} + \frac{K}{2}(x_{n+1} - x_n)^2 \right]$$

where $K = m\omega^2$, $m_n = m$ if n is even and $m_n = M > m$, if n is odd. Find the speed of sound in the chain, using the Laplace formula

$$c^2 = \frac{\partial P}{\partial \rho}$$

where P is the pressure and ρ is the density. Note that in 1D, pressure and force are the same thing. Compare your result with that obtained in the first lecture.

2. Consider a quantum chain of oscillators (see also your lecture notes) described by the Hamiltonian (1) above, but with x_n and p_n understood as quantum-mechanical operators and $m_n = m = M$. Diagonalize the Hamiltonian by quantizing the classical normal modes.

I.e., do the following: First, following your lecture notes from the first lecture, solve the classical problem (with $m = M$) and find the classical normal modes (do <u>not</u> simply take the limit of $m \to M$ in the more general problem we solved, but repeat all calculations in the simpler case of one type of atom in an elementary cell). Associate with each classical normal mode an appropriate set of quantum operators and write down the "diagonal" Hamiltonian (and associated commutation relations of the operators involved). Compare your result with the one obtained in the lectures using the Bogoliubov transform.

3. Using Bogoliubov transformations, diagonalize the following <u>**fermion**</u> Hamiltonian ($J_{1,2}$ and B are some constants):

$$\hat{\mathcal{H}} = \sum_{n=-\infty}^{+\infty} \left[J_1 \hat{f}_n^\dagger \hat{f}_{n+1} + J_2 \hat{f}_n \hat{f}_{n+1} - B \hat{f}_n^\dagger \hat{f}_n + \text{H. c.} \right]$$

This Hamiltonian appears in the context of a one-dimensional quantum magnetic (specifically the XY-model, as discussed in class). Find the spectrum of quasiparticles, $\varepsilon(k)$ of this Hamiltonian. Note that for $J_1 = J_2$ and $B = 0$ the dispersion disappears. Can this fact be understood without calculations?