Final Homework - PHYS 625 - Spring 2017
Deadline: Wednesday, May 10, 2017, in class

Professor Victor Galitski
Office: PSC 2270 and Toll Physics Bld. 2330

Web page: http://terpconnect.umd.edu/~galitski/PHYS625/

Do not forget to write your name, the homework number and staple your pages together!

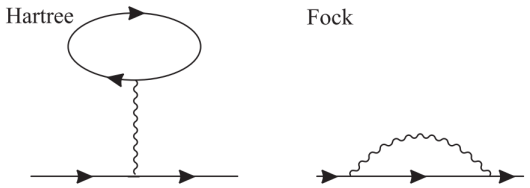
Green's function, Rashba spin-orbit coupling, Hartree-Fock diagrams, RPA screening, Berry phase

1. Electrons in a two-dimensional quantum well are often described by the Rashba model, defined by the following Hamiltonian

$$\hat{H}_{\text{Rashba}} = \sum_{\mathbf{p},\sigma,\sigma'} \left(\frac{\mathbf{p}^2}{2m} \delta_{\sigma,\sigma'} + \alpha \mathbf{e_z} \cdot [\boldsymbol{\tau}_{\sigma,\sigma'} \times \mathbf{p}] \right) \hat{a}^\dagger_{\mathbf{p},\sigma} \hat{a}_{\mathbf{p},\sigma'} \tag{1}$$

where α is a constant (spin-orbit coupling), $\mathbf{e_z}$ is a unit vector in the z-direction, and $\boldsymbol{\tau}_{\sigma,\sigma'}$ is a vector of Pauli matrices. Note that $\mathbf{p}$ is a two-dimensional vector. Diagonalize the Hamiltonian and find the Green's function for the non-interacting Rashba electrons.

2. Consider a three-dimensional system of fermions, which interact with each other with a point-like potential $V(\mathbf{r} - \mathbf{r}') = u_0 \delta(\mathbf{r} - \mathbf{r}')$. Calculate the fermionic self-energy in the Hartree-Fock approximation. *I.e.*, calculate the following diagrams.



Within the Hartree-Fock approximation, derive the general formula for the correction to the chemical potential in terms of the spin s, the fermion density n, and the interaction strength u_0. Note that in the model of "spinless fermions," the correction vanishes. Can this fact be understood without calculations?
Hint: Note that the integral of the Green's function over energy is simply the (Fermi) distribution function.

3. Consider a three-dimensional system of fermions interacting with each other via a long range potential $v(r) = g^2/r^2$ where r is the distance between the particles and g is a small constant. Doing the standard RPA perturbation theory ("bubble diagrams") find the screened potential in real space and calculate the spectrum of collective modes.
Hint: This problem is very similar to problem 3 (screening of Coulomb interactions in two dimensions) of Homework 3.

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4. Consider a localized spin-1/2 (quantum two-level system) subjected to a periodic-in-time magnetic field

$$\mathbf{b}(t) = (b_1 \cos(\omega t), b_1 \sin(\omega t), b_0), \tag{2}$$

where ω is the frequency with which the magnetic field rotates around the z-axis. The corresponding Hamiltonian is given in terms of Pauli matrices $\hat{\boldsymbol{\sigma}} = (\sigma_x, \sigma_y, \sigma_z)$:

$$\hat{H} = \mu \mathbf{b}(t) \cdot \hat{\boldsymbol{\sigma}}. \tag{3}$$

You are to find the Berry phase accrued during one full period $T = 2\pi/\omega$ of magnetic field rotation using (1) the geometric formula and (2) the exact solution to the problem and taking the appropriate adiabatic limit.

(a) Find the instantaneous eigenstates, $|\uparrow, \downarrow; t\rangle$ corresponding to (3) and pick one (e.g., $|\uparrow; t\rangle$) to follow in the course of adiabatic evolution.

(b) Calculate the dynamical phase and Berry phase using the formula $\gamma = i \int_0^T dt \, \langle \Psi(t)| \, \partial_t \, |\Psi(t)\rangle$.

(c) Find the exact time-evolution operator $\hat{U}(t)$ for (3) by following your lecture notes or the book https://www.amazon.com/Exploring-Quantum-Mechanics-Collection-Researchers/dp/0199232725/.

(d) Assume the system is initially in the ground state $|\Psi(0)\rangle = |\uparrow; 0\rangle$. Using the time-evolution operator find the the spinor $|\Psi(T)\rangle$ after one full period.

(e) Expand $|\Psi(T)\rangle$ obtained in (d) in terms of small ω and find the Berry phase acquired by the wave-function in the adiabatic limit (modulo 2π). Compare your result with the answer in part **(b)**.