Homework #1 — PHYS 625 — Spring 2016 Professor Victor Galitski
Deadline: Monday, February 22, 2016, in class Office: 2330 Physics

Relevant textbook: Abrikosov, A.A., Gor'kov, L.P., and Dzyaloshinskii, I.Ye., *Methods of Quantum Field Theory in Statistical Physicss*, Dover Publications Inc, New York, ISBN-10: 0486632288

Web page: `http://terpconnect.umd.edu/~galitski/PHYS625/`

Do not forget to write your name and the homework number!

Collective modes. Phonons. Bogoliubov transform

1. Consider a classical chain of oscillators (see also your lecture notes)

$$\mathcal{H} = \sum_{n=-\infty}^{\infty} \left[\frac{p_n^2}{2m_n} + \frac{K}{2} \left(x_n - x_{n+1} \right)^2 \right], \tag{1}$$

where $K = m\omega_0^2$, $m_n = m$ if n is even and $m_n = M > m$, if n is odd. Find the speed of sound in the chain, using the Laplace formula

$$c^2 = \frac{\partial P}{\partial \rho},$$

where P is the pressure and ρ is the density. Note that in 1D, pressure and force are the same thing.

2. Consider a quantum chain of oscillators (see also your lecture notes) described by the Hamiltonian (1) above, but with $\hat{x}_n$ and $\hat{p}_n = -i\hbar \frac{d}{dx_n}$ understood as quantum-mechanical operators and $m_n \equiv m = M$. Diagonalize the Hamiltonian using ladder operators in real and Fourier space and the Bogoliubov transformation. I.e., follow your lecture notes, but work out and present all technical details of the calculations.

3. Using Bogoliubov transformations, diagonalize the following **fermionic** Hamiltonian ($J_{1,2}$ and B are some constants):

$$\hat{\mathcal{H}} = \sum_{n=-\infty}^{\infty} \left(J_1 \hat{a}_n^\dagger \hat{a}_{n+1} + J_2 \hat{a}_n \hat{a}_{n+1} - B \hat{a}_n^\dagger \hat{a}_n + \text{ h. c. } \right).$$

This Hamiltonian appears in the context of a one-dimensional quantum magnetic (specificaly the XY-model, as discussed in class). Find the spectrum of quasiparticles $\varepsilon(k)$ of this Hamiltonian. Note that for $J_1 = J_2$ and $B = 0$ the dispersion disappears. Can this fact be understood without calculations?