This quiz contains examples from previous years exams. Working on solving these problems might help you prepare for the mid-term exam.

Problem 1.
For each of the operators listed in the left column find an eigenfunction (if any) from the list of functions in the column on the right and calculate the corresponding eigenvalues. Keep in mind that there could be more than one eigenfunction per operator, and also that the same function could be an eigenfunction for more than one operator.

	Operators
1	$\hat{p}_x$
2	$\hat{J}_z$
3	$\hat{J}_z^{\,2}$
4	$\hat{p}_x^{\,2} + 4\hat{x}^2$

	Functions
A	$e^{-\phi^2}$
B	$\sin(a\phi)e^{-x/b}$
C	$e^{x^2/\hbar}$
D	$e^{-i(5kx-3\phi)}$

Problem 2.
Consider a diatomic molecule, HCl, in a gas phase. Assume that there is no coupling between the vibrational and rotational states, so that the total Hamiltonian is the sum of the Hamiltonians corresponding to the rotational and vibrational motions. As a reminder, the atomic mass of chlorine is approximately 35 times the mass of hydrogen.

A. Determine the wavelength of the infrared radiation corresponding to a pure vibrational transition between the ground state and the first excited vibrational energy level. Assume that vibrations of the molecule can be modeled by the harmonic oscillator. HCl has a force constant of 516 $N \cdot m^{-1}$ ($1\ N = 1\ kg \cdot m \cdot s^{-2}$).

B. Microwave radiation causing a transition from the ground state to the first excited rotational state of this molecule has a wavelength of 0.469 mm. Assuming that rotational states of this molecule can be represented by the particle-on-a-sphere model, determine the bond length in HCl. (*You should be able to do this one after the next lecture, on Tue. For the moment assume that rotations of this molecule can be described by the particle-on-a-ring model*).

Problem 3. Consider a particle of mass μ on a ring of radius r. The ring is in the x-y plane. The particle is prepared in a superposition state described by the wavefunction (not normalized): $\Psi = N\left[5e^{i\phi} + 2e^{-i3\phi} + e^{i5\phi}\right]\sqrt{\frac{1}{2\pi}}$. (*Hint*: you can save yourself time if you realize that the individual functions are orthonormal)

A. List all values of the z-projection of the angular momentum (i.e. J_z) that one can obtain in a single measurement in this state.

B. What are the probabilities of measuring the values of the angular momentum in question A?

C. What is the expectation value of the z-projection angular momentum (i.e. J_z)?

D. Calculate the average energy value that will be obtained as a result of multiple measurements.

Problem 4. Consider a particle in a 1-D box of length *L*. Answer the following questions:
A. What are the probabilities of finding the particle in the first quarter (i.e. from 0 to *L*/4) and in the third quarter (i.e. from *L*/2 to 3*L*/4) in the state with the quantum number n = 3.
B. What are the expectation values of x^2 and p^2 in this state?
(*A general comment:* When dealing with integrals containing $\sin^2\alpha$ or $\cos^2\alpha$ it is often useful to use the following relationships: $\sin^2\alpha = (1-\cos 2\alpha)/2; \cos^2\alpha = (1+\cos 2\alpha)/2$, and also $2\sin\alpha \cdot \cos\alpha = \sin 2\alpha$. Integrals containing a product of $\sin\alpha$ (or $\cos\alpha$) and an integer power of x can be calculated by applying *integration by parts* as many times as necessary.)

Problem 5. Which of the two molecules, N_2 or CO, has the higher wavelength for the transition
(A) between the ground state and the first excited vibrational state;
(B) between the ground state and the first excited rotational state;
Assume for simplicity that vibrations in these molecules can be described by the harmonic oscillator model, and their rotational states can be modeled as the states of a particle on a ring. Also, assume for simplicity that these molecules have the same bond length and the same spring constant.

Problem 6.
Imagine a particle confined to the surface of a vertical cylinder with the height *L* and radius *R*. The particle is free to move along the surface of the cylinder, i.e. the potential energy is zero everywhere on its surface, but there is an infinitely high potential energy outside the surface. Write an expression for the allowed energy values and the corresponding eigenfunctions of this particle.
(*Hint:* consider which modes of motion are independent of each other and use the simple QM models that we studied in class. It might require more than one model for this problem, and it might be convenient to orient the coordinate frame such that its axis is vertical (i.e. along the cylinder axis) and the origin is in the center of the bottom of the cylinder.)

Problem 7. What are the allowed energy levels for a system analogous to a harmonic oscillator in which $V(x) = kx^2/2$ for $x \geq 0$ and $V(x) = \infty$ for $x < 0$?
(*Hint:* Examine the effect of the new boundary conditions on the choice of allowed solutions Ψ found for the standard harmonic oscillator).