Lecture 8

Part 1: Uncertainty and Games with Incomplete Information.

Lecture Outline

- Different reasons for uncertainty in games.
- Decision-making under uncertainty.
- Representations of games with uncertainty about payoffs.
- Representations of Games with uncertainty about actions.
Different Sources Of Uncertainty

- Uncertainty may be “exogenous” or outside the game. (Determined by “nature”.)
- Examples are exogenous changes in weather, in demand, in market-wide economic conditions.
- Uncertainty may be “endogenous” or the consequence of strategic decisions.
- Examples are games with simultaneous moves, unobservable actions by workers, unobservable pricing by rivals.

Allocations of Uncertainty or Information

- In games, the type of uncertainty is important and so is the allocation of uncertainty.
- If uncertainty is exogenous, then all players may be uninformed or perhaps only some.
- Example: In some cases, all firms must make investment decisions before the strength of the market is known, or perhaps some firms have inside information about the market.
Allocations of Uncertainty or Information

- If uncertainty is endogenous, then at least one player (the player who made the unobserved strategic move) must be partially informed.
- Example: Even if an employer may not be able to observe how hard or how carefully an employee is working, surely the employee knows this.

An Economic Role Of Uncertainty

- Sometimes the presence of (exogenous) uncertainty creates a reason for a market to exist.
- Examples include risk sharing (pooling) and risk selling.
- Example 1: Two farmers faces exogenous risk of the following sort. When the weather is good (with probability 1/2), profits are $15,000, when bad, profits are $5,000.
- Farmers risks are “negatively correlated”: when farmer 1’s weather is bad, farmer 2’s is good.
An Economic Role Of Uncertainty

- Individually each farmer faces a “lottery” paying $15,000 or $5,000 with equal probability. The expected value of the lottery is $10,000.
- Each season, though, the farmers’ joint income is $20,000. Jointly they do not face any uncertainty.
- If they are risk averse, there is an incentive for them to pool their risks. If the season is bad for farmer 1, farmer 2 pays 1 $5,000 and vice versa.
- As long as risks are not perfectly positively correlated, there is always the possibility of pooling risks to reduce risk: Examples are Insurance policies, mutual funds, stock portfolios.

An Economic Role Of Uncertainty

- Some people or institutions can bear risk more easily than others. For example, large and wealthy employers may not be very risk averse, while employees may be very risk averse.
- Recall, a risk averse person who faces a risky will always be willing to take something less than the expected value (for certain) in exchange for the gamble.
- Example: One farmer has expected utility function of profits given by \( u(x) = x^5 \)
- Again, with probability 1/2, he gets $15,000 or $5,000.
An Economic Role Of Uncertainty

- The farmer’s expected utility before each season begins is \(0.5 \times (5,000)^{0.5} + 0.5 \times (15,000)^{0.5} = 96.6\).
- If a crop insurance firm was risk averse, it would be indifferent between 10,000 for certain and $5,000 and $15,000 each with probability 1/2.
- Suppose the firm offered the farmer $9,400 at the beginning of the season in return for a full claim on the farmer's profits.

The farmer receives \((9400)^{0.5} = 96.95 > 96.6\) and is better off. The firm gets a gamble which pays $10,000 on average in return for a payment of 9400 and is better off.
- Both parties have made an improvement.
- Note that in some sense, the farmer has become the employee of the firm.
- What if the farmer knows before selling the risk what the weather will be like? Should the firm pay $9400?
- What if the firm cannot see how hard the farmer works?
- The answers to these questions lead to a strategic analysis
Representing Uncertainty in Games

In the above game, an employer needs to decide whether to promote or fail an employee.

- Which is better depends on whether \( x > 50 \) or not.
- \( x > 50 \) means that the employee is a hard worker. If I promote her, she will work hard and earn me 80 while if I fail her, she will only earn me 60.
- \( x < 50 \) means she is a soft worker. If I promote her, she will earn me 30 while if I fail her I will get 50.

How do we represent this?
Representing Uncertainty in Games

- In this representation, Nature moves first and selects whether the employee is hardworking or lazy.
- Without directly observing Nature’s choice, the employer has to decide whether or not to promote her.
- Notice that as we would expect, the employee does know what Nature has selected.
Informational Incentives in Games

- In situations where some players have access to more information than others, strategic incentives to reveal or conceal arise.
- Suppose Player A is a better informed player. A may wish to
  1. Conceal information or reveal misleading information
  2. Reveal only selective information
- If B is less well-informed, B may wish to
  1. Elicit information
  2. Remain ignorant

Whatever the decision, note that generally it is not enough simply to ask someone the truth.
- When strong incentives to mislead exist, other more reliable devices will be needed.
- “Actions speak louder than words” and we often need to find ways to induce players to “act” in a way that reveals their information.
Informed players who wish to find credible ways to reveal their information engage in “signaling” behavior.

Informed players who wish to conceal their information engage in “signaling jamming” behavior.

Uninformed players who wish to elicit behavior, engage in “screening” techniques.