Lecture Six

Market Games in Quantities and Prices

Outline of Lecture:
Quantities

- Description of market, perfectly competitive outcome and monopoly outcome.
- Description of duopoly competition in quantities (Cournot)
- Computation of Best Response
- Finding equilibrium using Best Responses.
- Comparison of Cournot Equilibrium with perfect competition and monopoly.
- Graphical Approach
- Adding more firms.
Cournot Competition: The Market

Market Price, \( P = 130 - Q \) when \( Q \leq 130 \)
\[ = 0 \] otherwise

Market Quantity, \( Q = x_1 + x_2 + \ldots + x_n = \sum x_i \)

Quantity vector, \( \mathbf{x} = (x_1, x_2, \ldots, x_n) \)
here \( x_i \) represents firm i’s quantity delivered to the market

\[ \therefore \] For a market with 2 firms,
\[ Q = x_1 + x_2 \quad \text{and} \quad \mathbf{x} = (x_1, x_2) \]

Constant marginal cost = \( c \)

Perfectly Competitive Equilibrium

- Market price equals marginal cost
- In this market, marginal cost = \( c = $10 \)
- \( Q = 130 - P = 130 - 10 = 120 \)
- With \( n \) firms:
- \( \mathbf{x}^* = (120/n, 120/n, \ldots, 120/n) \)
- Profit for any firm = \( (10 - 10) \times x_i \)
  \[ = 0 \]
Monopoly Equilibrium

Market profits are as large as possible

A monopoly will maximize total market profit, \( \Pi = (P - c) Q \)

\[ \Rightarrow \quad \Pi = (120 - Q) Q \]

For maximizing profit, marginal profit of producing one more unit needs to be zero

\[ \Rightarrow \quad 0 = \frac{\partial \Pi}{\partial Q} = 120 - 2Q \]

\[ \therefore \quad Q^* = 60 \quad \text{and} \quad P^* = 70 \]

Total profits = \((120-60) \times 60 = \$3600\)

Cournot Equilibrium

In a perfectly competitive market, all firms select \( x \) taking \( P \), market price as given.

In a monopoly, the single firm selects \( x \) recognizing that \( P \) will change with \( x \) according to the market demand curve.

In a Cournot world, each firm \( i \) chooses \( x_i \) simultaneously. They treat all other firms' choice of \( x_j \) as given.

Their choice of \( x_i \) will affect price.
Cournot competition for two firms: A firm’s profits

Firm $i$’s profits:

$\Pi_i(x) = \text{revenue} - \text{cost}$

$= Px_i - cx_i$

$= (P - c)x_i$

$\therefore \Pi_1(x) = (P - c)x_1$ and

$\Pi_2(x) = (P - c)x_2$

Cournot Competition: Two Firms

- Quantity as a continuous strategy
- Finding Cournot equilibria using the calculus
- The best response function
Cournot Competition, two firms, Deriving a Firm’s Best Response

Profit function of firm $i$: \( \Pi_i(x) = (P - c)x_i \)

Consider firm 1

Firm 1 maximizes its profit by producing up to the point where marginal profit equals zero:
\[
0 = \frac{\partial \Pi_1}{\partial x_1} = (P - c) + x_1 \frac{\partial P}{\partial x_1} \\
\Rightarrow \quad 0 = (120 - x_1 - x_2) + x_1(-1) \\
\Rightarrow \quad 0 = 120 - 2x_1 - x_2 \quad \text{Or} \\
(120 - x_2)/2 = x_1
\]

Cournot Competition, two firms, Deriving a Firm’s Best Response

Similar computations for Firm 2 yield
\[
(120 - x_1)/2 = x_2
\]

In finding a Nash Equilibrium, we are looking for a pair \((x^*_1, x^*_2)\) such that \(x^*_1\) is a best response for Firm 1 to \(x^*_2\) and \(x^*_2\) is a best response for Firm 2 to \(x^*_1\).
Cournot Competition, two firms, Solving for the equilibrium

This means that \((x^*_1, x^*_2)\) must solve both
\[
\frac{120 - x^*_1}{2} = x^*_2 \quad \text{AND} \\
\frac{120 - x^*_2}{2} = x^*_1
\]
We can plug in \(x^*_1\) from equation 2 into equation 1 to solve for \(x^*_2\). The solution gives us:

Cournot Equilibrium:
\[x^*_2 = x^*_1 = x^* = (40, 40)\]
Notice that total market quantity is 80 and price is \(130-80=50\).
Each firm earns profits, \((50-10)*40=1600\)
Total industry profits are $3200
Cournot equilibrium in the market

- Monopoly is associated with the highest price, lowest quantity, and highest profit
- Perfect Competition is associated with the lowest price, highest quantity, and zero profit
- Cournot equilibrium lies in between on all three dimensions

Cournot Competition, Two Firms, Many Strategies

*Cournot competition between two firms leads to an outcome between monopoly and perfect competition*
Cournot equilibrium in the market

Finding Cournot best responses

- Firm 1’s first-order condition is:
  \[ 2x_1 + x_2 = 120 \]

- Solving for \( x_1 \) as a function of \( x_2 \) yields firm 1’s best-response function:
  \[ x_1 = f_1(x_2) = 60 - \frac{x_2}{2} \]

- Similarly, firm 2’s best-response function:
  \[ x_2 = f_2(x_1) = 60 - \frac{x_1}{2} \]
Cournot best responses, \( x^* = \text{Cournot equilibrium} \)

\[
x_2 = f_2(x_1) = 60 - x_1/2
\]

\[
x_1 = f_1(x_2) = 60 - x_2/2
\]

**Cournot Variations, Including Many Firms**

- Cost advantage translates into market share advantage
- The Cournot limit theorem: the higher the number of firms, the closer Cournot equilibrium gets to perfect competition
- The Cournot limit is good for the economy
Cournot variations, including many firms

- For any firm, profit \( \Pi_i(x) = (P - 10)x_i \)
- Since all firms face the same costs and sell identical products, the game is symmetrical. The profit maximization strategy for all the firms will be the same.
- We will focus on firm 1 to derive the Cournot equilibrium

Firm 1 wants to maximize profit, 
\[ \Pi_1(x) = (P - 10)x_1 \]

Firm 1 maximizes profit when
\[ 0 = \frac{\partial \Pi_1}{\partial x_1} = (P - 10) + x_1(\partial P/\partial x_1) \]
\[ = 120 - \sum x_i - x_1 \]

By Symmetry, \( \sum x_i = nx_1 \)
\[ \therefore 0 = 120 - (n+1)x_1 \]
\[ \Rightarrow x_1^* = \frac{120}{n+1} \]
Cournot variations, including many firms

- Market Quantity,
  \[ Q^* = \sum x_i = nx_i = \frac{120n}{n+1} \]
and Market Price,
\[ P^* = 130 - Q^* = 130 - \left[ \frac{120n}{n+1} \right] \]

- \( n \to \infty \Rightarrow P^* = $10 \) and \( Q^* = 120 \)

- Cournot equilibrium becomes perfect competition equilibrium as \( n \) goes to infinity

Cournot Limit Theorem, surplus analysis: \( n = 1 \)

![Diagram showing surplus analysis for Cournot equilibrium with \( n = 1 \)]
Cournot Limit Theorem, surplus analysis: $n = 2$

Cournot Limit Theorem, surplus analysis: $n = \infty$
Is a Cournot Equilibrium Collusion?

- Cartels vs. Cournot equilibrium