Lecture Five

Mixed Strategies

Mixed Strategies: Outline

- What is a mixed strategy?
- When are they used in Nash Equilibrium? -- Motivation and Existence.
- Using Mixed Strategies to find Nash Equilibria in 2 by 2 games.
- Some examples. Chicken, Seles and Hingis again.
- Mixing means unpredictability.
What Are Mixed Strategies?

- For any given set of pure strategies, mixed strategies are rules that tell a player what percentage of the time to play each pure strategy.
- Players use these rules every time they play.
- Each play, though, the play should be unpredictable. (Thus it is possible that a rule which says “Up 50% of the time” has as its outcome: {Up, Up, Up, Up,...} though of course the more Up’s there are the less likely it will occur.)

When Are Mixed Strategies Used?

- Mixed strategies are needed when no pure strategy equilibria exist in a game.
- Mixed strategies may generate equilibria even when pure strategy equilibria exist.
- They tend to emerge when a conflict of interest generates a “cycle” in a game.
When Are Mixed Strategies Used?

- For example, one player may wish to mimic the other while the other may wish to differentiate herself from the first player.
- They help to provide unpredictability.

Using Mixed Strategies: An Example

- Return to the Seles/Hingis tennis match.
- Seles wants to choose a different strategy from Hingis. Hingis wants to match Seles’s strategy.
- If one player knew for certain what the other would do, then the other player would want to change her strategy.
- Therefore, each needs to be unpredictable.
Seles Tries to Make Hingis Guess: A game with no Pure Strategy Equilibrium.

Using Mixed Strategies: An Example

- Focus on Hingis first. Suppose Seles chooses DL with probability $p$ and CC with probability $1-p$.
- Dixit and Skeath call this a $p$-mix.
- If she chooses this randomization, what will Hingis get for each pure strategy?
- If she chooses DL, she gets $50p + 10(1-p) = 10 + 40p$.
- If she chooses CC she gets $20p + 80(1-p) = 80 - 60p$. 
Using Mixed Strategies: An Example: Hingis

- Notice that for $p > .7$, $10 + 40p > 80 - 60p$.
- For $p < .7$, $10 + 40p < 80 - 60p$.
- At $p = .7$, $10 + 40p = 80 - 60p$.
- Therefore DL is her only BR for $p > .7$.
- CC is her only BR for $p < .7$.
- And \{DL, CC\} and any mixture between them is a BR for $p = .7$.
- If we let $q$ be the probability Hingis chooses DL, diagrammatically, her BR(p) looks like:

Best Response Curves and Mixed Strategy Equilibrium: Hingis
Using Mixed Strategies: An Example: Seles

• Focus on Seles. Suppose Hingis chooses DL with probability $q$ and CC with probability $1-q$. (a $q$-mix.)
• If she chooses this randomization, what will Seles get for each pure strategy?
  • If she chooses DL, she gets $50q + 80(1-p) = 80 - 30q$.
  • If she chooses CC she gets $90q + 20(1-q) = 20 + 70q$.
• Notice that for $q<.6$, $80 - 30q > 20 + 70q$.
• For $q>.6$, $80 - 30q < 20 + 70q$.
• At $q=.6$, $80 - 30q = 20 + 70q$.
• Therefore DL is her only BR for $q<.6$
• CC is her only BR for $q>.6$
• And {DL, CC} and any mixture between them is a BR for $q=.6$
• Diagrammatically, this looks like:
Mixed Strategies

- Putting the two together, we find the pair \((p^*, q^*)\) such that when Seles’s \(p\)-mix is .7 and Seles’s \(q\)-mix is .6 is a best response for Hingis (though not her only one).
- and, when Hingis’s \(q\)-mix is .6 \(p=.7\) is a best response for Seles (though not her only one).
Best Response Curves and Mixed Strategy Equilibrium

Computing Mixed Strategies

- In the Hingis/Seles game, it is evident that there is a unique equilibrium and it is in mixed strategies.
- However, there may also be mixed strategy equilibria in games where pure strategy equilibria exist as well.
- Consider the game of chicken from before.
The Game of Chicken.

2 Pure Strategy Nash Equilibrium in the Game of Chicken.

- Suppose that Dean chooses Swerve with probability $q$.
- James’ payoffs from Swerve and Down respectively are $-(1-q)$ and $3q-2$.
- The value of $q$ which makes James indifferent is $q^* = .5$.
- A similar argument gives $p^* = .5$ for Dean.
- This gives James’ BR graphically as
Multiple Mixed Strategy Equilibrium: James

Multiple Mixed Strategy Equilibrium: Dean
Surprising Effects in Games

- Sometimes the interactions of strategic behavior have surprising effects on outcomes.
- Return to the Hingis/Seles example. Suppose that Hingis becomes better at returning down the line shots so the payoff matrix.
- Will her percentage of DL choices go up?
Seles Tries to Make Hingis Guess: A game with no Pure Strategy Equilibrium.

Mixed Strategy Responses

- To compute mixed strategy equilibria, we need to find the $p^*$ which makes Hingis indifferent between DL and CC and the $q^*$ which makes Seles indifferent.
- Seles: $30q^* + 80(1-q^*) = 90q^* + 20(1-q^*)$
  $60 = 120q^*$, or $q^* = .5$.
- Hingis: $30p^* + 90(1-p^*) = 80p^* + 20(1-p^*)$
  $70 = 120p^*$ or $p^* = 7/12 = .58$. 
Mixed Strategy Equilibrium Changes When Hingis Improves Her Game

Hingis Improves Her Down the Line Defense and uses it less often.

- notice that now, $q^* = .5$ instead of .6.
- She utilizes it less often because Seles is now reluctant to return DL given her improvement.
- Nevertheless, this change has raised Hingis’s payoff in the game. Prove it!
Mixed Strategies: Final Comments

• When players randomize between pure strategies, they are actually indifferent as to which probability they choose, so why do they select the number we compute for them?
• Answer: This is the number which keeps the opponent off guard.

Mixed Strategies: Final Comments

• This highlights the importance of true randomness.
• If a player had a mixed strategy of DL with 70%. Suppose the game was played 10 times in a row and she always did DL the first 7 times and CC the next 3. Is this random?
• No! Unpredictability is important.
Mixed Strategies: Final Comments

• Computing mixed strategy equilibria is very similar to computing equilibria in games where strategies are continuous variables.
• We will see this next in the simultaneous version of market games.

Mixed Strategies: Final Comments

• We have seen that some simultaneous move games do not possess equilibria in PURE strategies.
• It is a general result, though, that ALL simultaneous move games with finite strategy choices possess equilibria in mixed strategies.
• In fact, the result is a little stronger. Typically, (though not always) these games possess an odd number of equilibria.
Mixed Strategies: Final Comments

• To see why, consider the diagram.
• For the vertical axis player, start from either 0 or 1 at the left side and draw some “step” diagram to get to 1 or 0 on the left side.
• For the horizontal axis player, start from 0 or 1 on the bottom and draw some “step” diagram to get to 1 or 0 on the top.
• Will the two lines intersect? How many times?