Lecture Three

Sequential Games

Sequential Games: Outline

- Simple examples of sequential games
- one-person decision problems: are they “Games”?
- Subgames
- Solving Games: Rollback or Backward Induction
- Threats, promises and credibility
- Sequential Market games - quantity or price competition
- First mover advantage?
- Ultimatum games
1-player Games with Perfect Information

- Perfect Information
- Extensive form of a game (tree diagram)
- Features of the extensive form
  - endpoints
  - nodes
  - information sets
  - branches
  - payoffs
- Solving a game by backward induction
- A strategy is a complete plan of auction

A One-Person “Game”

The Current You
- Med School
- Law School
- Bus School

Future You
- Comm Med
- Brain Surgery
- GP
- DA
- Politics
- Corporate
- Marketing
- Finance
- MGT

Payoffs:
- Comm Med: 50K
- Brain Surgery: 150K
- GP: 500K
- DA: 50K
- Politics: 35K-200K
- Corporate: 600K
- Marketing: 100K
- Finance: 250K
- MGT: 0-5M
Sequential Games

- recall the distinction between decision problems and strategic problems.
- my simple choice of the best coffee shop to go to is a decision problem.
- my (hard) choice of the best coffee shop to go to in order to avoid my boss is a strategic problem (I need to anticipate her choice in order to decide optimally.)

Sequential Games

- strategic problems are the focus of game theory and, as we will see, can be very complicated.
- sequential games with perfect information are the simplest type of strategic problem.
- this is because they are really just a sequence of individual decision problems.
- they are often described in “game trees”.
Telex vs. IBM, extensive form: subgame, perfect information

Subgame

Enter

Smash

1, 5

Accommodate

2, 2

0, 0

Example of a sequential game with perfect information

- Telex is considering entering the computer business.
- notice that in order for Telex to determine its best strategy, it must be able to anticipate IBM’s response.
- How should it approach this problem?
Telex vs. IBM, extensive form: noncredible equilibrium

Subgames and their equilibria

- The concept of subgames
- Equilibrium of a subgame
- Credibility problems: threats and promises you have no incentives to carry out when the time comes
- Two important examples
  - Telex vs. IBM
  - Centipede
Telex vs. IBM, extensive form: credible equilibrium

Centipede, extensive form
Centipede, extensive form

Maintaining Credibility via Subgame Perfection

- Subgame perfect equilibria: play equilibria on all subgames
- They only make threats and promises that a player does have an incentive to carry out
- Subgame perfection as a necessary condition for solution of games in extensive form
Rollback Equilibrium  
(Look Ahead and Reason Back)

- This is also called Backward Induction
- Backward induction in a game tree leads to a subgame perfect equilibrium
- In a subgame perfect equilibrium, “best responses” are played in every subgame

Credible Threats and Promises

- The variation in credibility when money is all that matters to payoff
- Telex vs. Mean IBM
- Centipede with a nice opponent
- The potential value of deceiving an opponent about your type
Telex vs. Mean IBM

Centipede with a nice opponent, extensive form
Conscription, extensive form

Conscription, \( b = \$300 \) and \( c = \$400 \)
Credible Quantity Competition: Stackelberg Equilibrium

- The first mover advantage in Stackelberg competition
- One firm sends its quantity to the market first. The second firm makes its moves subsequently.
- The strategy for the firm moving second is a function
- Incredible threats and imperfect equilibria

Stackelberg Equilibrium for two firms

Market Price, \( P = 130 - Q \)

Market Quantity, \( Q = x_1 + x_2 \)

Constant average variable cost, \( c = $10 \)

Firm 1 ships its quantity, \( x_1 \), to market first

Firm 2 sees how much firm 1 has shipped and then ships its quantity, \( x_2 \), to the market
Stackelberg Equilibrium: Firm 2 wants to maximize its profits **given Firm 1’s choice.**

Firm 2’s profit function is given by:

\[ u_2(x_1, x_2) = [130 - x_1 - x_2 - 10] x_2 \]

Differentiate wrt \( x_2 \) gives:

\[ 0 = (120 - x_1 - 2x_2) \]

or

\[ (120 - x_1)/2 = x_2 \]

Stackelberg Equilibrium: Firm 2 wants to maximize its profits **given Firm 1’s choice.**

- This gives us Firm 2’s Best response given Firm 1’ choice of \( x_1 \)
- We can write this as a function of \( x_1. \)
- \( g(x_1) = (120 - x_1)/2 \)
Stackelberg Equilibrium: Firm 1 also wants to maximize its profits

Firm 1’s profit function is given by:
\[ u_1(x) = [130 - x_1 - g(x_1) - 10] x_1 \]

Substituting \( g(x_1) \) into that function:
\[ u_1(x) = (120 - x_1 - 60 + x_1/2) x_1 \]

\[ \therefore \text{Firm 1’s profits depend only on its shipment} \]

Taking the first order condition for \( u_1(x) \):
\[ 0 = 60 - x_1 \]

The Stackelberg Equilibrium for two firms

The Stackelberg equilibrium value of firm 1’s shipments, \( x_1^* = 60 \)

Firm 2’s shipments, \( x_2^* = 60 - 60/2 = 30 \)

Market Quantity, \( Q = 60 + 30 = 90 \)

Market Price, \( P = 130 - 90 = $40 \)

We will see that this equilibrium is different from Cournot competition’s equilibrium, where \( x_1^* = x_2^* = 40 \), \( Q = 80 \) and \( P = $50 \) (Q’s are chosen simultaneously.)
Credible Price Competition: Bertrand-Stackelberg Equilibrium

- Firms use prices as the strategic instrument
- The strategy for the firm moving second is a function
- Firm 2 has to beat only firm 1’s price which is already posted
- The second mover advantage in Bertrand-Stackelberg competition

Bertrand-Stackelberg Equilibrium for two firms

Market Price, \( P = 130 - Q \) and
Constant average variable cost, \( c = $10 \)

Firm 1 first announces its price, \( p_1 \)

Firm 2’s profit maximizing response to \( p_1 \):

\[
\begin{align*}
    p_2 &= 70 & \text{if } p_1 \text{ is greater than } 70 \\
    p_2 &= p_1 - 0.01 & \text{if } p_1 \text{ is between } 70 \text{ and } 10.02 \\
    p_2 &= p_1 & \text{if } p_1 = 10.01 \\
    p_2 &= 10 & \text{otherwise}
\end{align*}
\]

Get competitive outcome; no extra profits!
Differentiated Products

• Product differentiation mutes both types of mover advantage
• A mover disadvantage can be offset by a large enough cost advantage

Two firms in a Bertrand-Stackelberg competition

The demand function faced by firm 1:
\[ x_1(p) = 180 - p_1 - (p_1 - \text{average } p) \]
\[ \Rightarrow x_1 = 180 - 1.5p_1 + 0.5p_2 \]

Similarly, the demand function faced by firm 2:
\[ x_2 = 180 + 0.5p_1 - 1.5p_2 \]

Constant average variable cost, \( c = $20 \)
Two firms in a Bertrand-Stackelberg competition:

**Equilibrium prices**

Knowing that firm 2 will determine $p_2$ by using $g(p_1)$, firm 1 tries to maximize its profit:

$$\text{max} \ (p_1 - 20)[180 - 1.5p_1 + 0.5(70 + p_1/6)]$$

Profit maximizes when the first order condition is satisfied:

$$0 = 215 - (17/12)p_1 + (p_1 - 20)(-17/12)$$

$$\therefore \ p_1^* = 2920/34 = 85.88$$

Firm 2, which moves last, charges slightly lower price than $p_1^*$:

$$p_2^* = 70 + p_1^*/6 = 70 + 14.31 = 84.31$$

Two firms in a Bertrand-Stackelberg competition: Profits for the two firms

Firm 1 sells less than firm 2 does:

$$x_1^* = 93.34 \quad \text{and} \quad x_2^* = 96.48$$

Firm 1’s profit, $u_1^* = (93.34)(85.88 - 20) = 6149.24$

Firm 2’s profit, $u_2^* = (96.48)(84.31 - 20) = 6204.63$

Firm 2, the second mover, makes more money
This Offer is Good for a Limited Time Only

- The credibility problems behind the marketing slogan
- The principle of costly commitment
- Industries where the slogan is credible

An example of “This offer is good for a limited time only”

- Exploding job offers
  - An early job offer with a very short time to decide on whether to take the job.
  - Risk-averse people often end up accepting inferior job offers
Is it always better to move first?

- it is in the Stackelberg game.
- is it in the Bertrand game?
- what about the cut the cake game?

Cut the Cake

![Cut the Cake Diagram]

1

2

(0,1)

(1,0)

(1/3,2/3)

(2/3,1/3)

(3/8,5/8)

(5/8,3/8)

(7/16,9/16)

(9/16,7/16)
Ultimatum Games

- consider the following game
- I want to buy a car from a seller.
- I know the seller can always sell the car to a used car lot for $1000.
- I value the car at $1500. The seller knows this.
- I need to leave the country on business at the end of the day and have time to make only one offer. The seller knows this as well.
- I can make offers in $1 dollar increments.
- What is the rollback equilibrium of this bargaining game?

Ultimatum Games: Analysis

- This is an example of a take-it-or-leave-it bargaining game.
- What is a subgame? A subgame starts when the seller is in the position to accept or reject my price offer.
- When the seller has a price offer of $1200 on the table, this represents a different subgame than when an offer of $1199 is on the table. Therefore, there are as many possible subgames as there are prices.
- The only interesting ones are those with prices below $1500 and above $1000 so there are 500 different possible subgames.
Ultimatum Games: Solution

• Suppose that I have offered \( p \) between $1000 and $1500. \( p \) represents the subgame.
• Now the seller must decide Accept or Reject.
• If she accepts, she gets \( p-1000 \) (over and above what her next best alternative is)
• If she rejects, she gets 0 (over and above what her next best alternative is)
• Conclusion: If \( p>1000 \) she should accept.

Ultimatum Games: Solution (ctd)

• Now, what is my best strategy.
• I know that for every \( p>1000 \) I offer, seller will accept, therefore, my payoff is \( 1500-p \) if \( p>1000 \) and 0 if \( p<1000 \). (equality is indeterminate)
• my best response is to offer \( p=1001 \). The seller should accept, she gets 1 and I get 499.
• This is quite general, in one shot take-it-or-leave-it games, the offeror gets (almost) all the surplus.
• first mover advantage here is very strong.
Appendix. Ultimatum Games in the Laboratory

- Games with take-it-or-leave-it structure
- In experiments, subjects playing such games rarely play subgame perfect equilibria
- The nice opponent explanation vs. the expected payoff explanation

Modifications

- More players? Easy, as long as we remember to rollback
- More than two strategies? We have already done it. (Cut the cake, (4 choices for player 1), Stackelberg infinite choices.)
- Bottom line. Sequential games with perfect information are interesting but pretty easy.